Critical analysis of the effective delayed-neutron fraction calculation routes by deterministic and Monte Carlo codes.

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Layout

- ✓ Introduction
- ✓ Theoretical analysis
- ✓ Standard and Monte Carlo β_{eff} formulations
- ✓ ERANOS formalism and calculation set up
- ✓ ERANOS results
- ✓ Conclusions

Introduction

- The increasing use of Monte Carlo codes in the field of nuclear reactors calculation and the studies on ADS have renewed the interest in the theoretical and computational evaluation of the main integral parameters characterizing subcritical systems.
- Some particular parameters, as the effective delayed neutron fraction, are evaluated in Monte Carlo codes by formulations which do not require the calculation of the adjoint flux.
- The assessment of the various formulations of the effective delayed neutron fraction is crucial for the system evaluation, since it plays an important role in determining its dynamic characteristics.
- This work is focused on a theoretical and computational analysis about how the different β_{eff} definitions are connected.

Introduction

- Theoretical results show how the Monte Carlo formulation of β_{eff} may be connected to the classical definition, interpreting the classical one through a reactivity evaluation based on an "improved" first order approach of perturbation theory.
- The computational analysis is carried out in a coherent and consistent way, using the same deterministic code (ERANOS) and neutron data library (JEFF 3.1) for the β_{eff} evaluation.
- The GUINEVERE system is selected as a relevant test case for ADS technology. The GUINEVERE experience, mainly devoted to the issues concerning on-line reactivity monitoring in ADS, will be performed by using a modified lay-out of the VENUS critical facility located at the Belgium SCK•CEN Mol-site, coupling the subcritical core facility to a deuteron accelerator delivering, by a continuous or pulsed beam, 14 MeV neutrons by deuterium-tritium reactions.

Theoretical analysis

Let us take as *reference* system the following eigenvalue problem:

$$\mathbf{L}\varphi = \omega\mathbf{F}\varphi$$

With **L** loss operator, **F** fission operator and ω=1/k.

If a perturbation δ**F** is introduced:

$$\mathbf{L}\varphi^{*} = \left[\omega + \delta\omega\right]\left[\mathbf{F} + \delta\mathbf{F}\right]\varphi^{*}$$

Theoretical analysis

Following the exact approach of Perturbation Theory (PT), i.e. considering the adjoint problem corresponding to the *reference* system:

$$\mathbf{L}^{+}\varphi^{+} = \omega\mathbf{F}^{+}\varphi^{+}$$

we obtain:

$$\frac{\delta\omega}{\omega+\delta\omega} = 1 - \frac{k^{*}}{k} = -\frac{<\varphi^{+}\delta\mathbf{F}\varphi^{*}>}{<\varphi^{+}\mathbf{F}\varphi^{*}>}$$

Theoretical analysis

If we follow the first order PT approach, *while retaining the second order term δωδ**F**φ*, we obtain the "improved" first order formulation:

$$\frac{\delta\omega}{\omega+\delta\omega}=1-\frac{k^{*}}{k}\cong-\frac{<\varphi^{+}\delta\mathbf{F}\varphi>}{<\varphi^{+}\mathbf{F}\varphi>}$$

If the perturbation is δ**F**=-$\mathbf{F_d}$, with $\mathbf{F_d}$ as delayed neutrons fission operator, the perturbed system can be written as:

$$\mathbf{L}\varphi_{p}=\omega_{p}\left[\mathbf{F}-\mathbf{F_d}\right]\varphi_{p}$$

with the perturbed flux given by the prompt flux φ_p and $\omega_p=1/k_p$.

Theoretical analysis

We have:

$$1-\frac{k_p}{k}=\frac{<\varphi^{+}\mathbf{F_d}\varphi_p>}{<\varphi^{+}\mathbf{F}\varphi_p>}\cong\frac{<\varphi^{+}\mathbf{F_d}\varphi>}{<\varphi^{+}\mathbf{F}\varphi>}=\beta_{eff}$$

PT "improved" first order formulation

The effective delayed neutron fraction β_{eff} is an "*improved*" PT first order formulation of the formula $1-(k_p/k)$, widely used in Monte Carlo codes as β_{eff} estimator.

Standard and Monte Carlo β_{eff} formulations

Effectiveness of delayed neutrons:

$$\beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{<< \varphi^+(\mathbf{r},E)\,\chi_{d,i}^{(m)}(E) < \Sigma_f^{(m)}(\mathbf{r},E')\varphi(\mathbf{r},E') >_{E'} >_E >_{\mathbf{r}}}{\sum_m << \varphi^+(\mathbf{r},E)\,\chi^{(m)}(E) < \nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\,\varphi(\mathbf{r},E') >_{E'} >_E >_{\mathbf{r}}}$$

Fissile isotope

Delayed neutron group

Average value of delayed neutrons

Delayed neutron spectrum

$$\boldsymbol{\Omega}\cdot\nabla\varphi(\mathbf{r},E)+\Sigma_t(\mathbf{r},E)\varphi(\mathbf{r},E)=\frac{1}{k}\sum_m \chi^{(m)}(E) < \nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi(\mathbf{r},E') >_{E'}$$
$$+ < \Sigma_s(\mathbf{r},E'\rightarrow E)\varphi(\mathbf{r},E') >_{E'}$$

$$-\boldsymbol{\Omega}\cdot\nabla\varphi^+(\mathbf{r},E)+\Sigma_t(\mathbf{r},E)\varphi^+(\mathbf{r},E)=\frac{1}{k}\sum_m \nu^{(m)}(E)\Sigma_f^{(m)}(\mathbf{r},E) < \chi^{(m)}(E')\varphi^+(\mathbf{r},E') >_{E'}$$
$$+ < \Sigma_s(\mathbf{r},E\rightarrow E')\varphi^+(\mathbf{r},E') >_{E'}$$

Theoretical analysis

Current Monte Carlo (MC) calculations approximate β_{eff} by:

$$\beta_{eff}^{MC} = 1 - \frac{k_p}{k},$$

$$\boldsymbol{\Omega}\cdot\nabla\varphi_p(\mathbf{r},E)+\Sigma_t(\mathbf{r},E)\varphi_p(\mathbf{r},E)=$$

$$\frac{1}{k_p}\sum_m\left[\chi^{(m)}(E)<\nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E')>_{E'}-\sum_i\chi_{d,i}^{(m)}(E)\overline{\nu}_{d,i}^{(m)}<\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E')>_{E'}\right]$$

$$+<\Sigma_s(\mathbf{r},E'\to E)\varphi_p(\mathbf{r},E')>_{E'}$$

perturbed

$$\boldsymbol{\Omega}\cdot\nabla\varphi(\mathbf{r},E)+\Sigma_t(\mathbf{r},E)\varphi(\mathbf{r},E)=\frac{1}{k}\sum_m\chi^{(m)}(E)<\nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi(\mathbf{r},E')>_{E'}$$

$$+<\Sigma_s(\mathbf{r},E'\to E)\varphi(\mathbf{r},E')>_{E'}$$

unperturbed

Theoretical analysis

We have:

$$\textbf{(1)}\quad \beta_{eff}^{MC} = 1 - \frac{k_p}{k} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{<< \varphi^+(\mathbf{r},E)\chi_{d,i}^{(m)}(E) < \Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E') >_{E'} >_E >_{\mathbf{r}}}{\sum_m << \varphi^+(\mathbf{r},E)\chi^{(m)}(E) < \nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E') >_{E'} >_E >_{\mathbf{r}}}$$

"*improved*" PT first order formulation

$$\textbf{(2)}\quad \beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{<< \varphi^+(\mathbf{r},E)\chi_{d,i}^{(m)}(E) < \Sigma_f^{(m)}(\mathbf{r},E')\varphi(\mathbf{r},E') >_{E'} >_E >_{\mathbf{r}}}{\sum_m << \varphi^+(\mathbf{r},E)\chi^{(m)}(E) < \nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi(\mathbf{r},E') >_{E'} >_E >_{\mathbf{r}}}$$

*The classical β_{eff} definition given in **(2)** is an "improved" PT first order approximation of the relationship 1-(k_p/k) given in **(1)**.* The coherent prompt flux φ_p to be used in the β_{eff} definition given in **(1)** is the eigenfunction of:

$$\boldsymbol{\Omega}\cdot\nabla\varphi_p(\mathbf{r},E) + \Sigma_t(\mathbf{r},E)\varphi_p(\mathbf{r},E) =$$

$$\frac{1}{k_p}\sum_m \left[\chi^{(m)}(E) < \nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E') >_{E'} - \sum_i \chi_{d,i}^{(m)}(E)\bar{\nu}_{d,i}^{(m)} < \Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E') >_{E'} \right]$$

$$+ < \Sigma_s(\mathbf{r},E'\to E)\varphi_p(\mathbf{r},E') >_{E'} .$$

Theoretical analysis

The other way round. If we calculate k_p and φ_p by:

$$\boldsymbol{\Omega}\cdot\nabla\varphi_p(\mathbf{r},E)+\Sigma_t(\mathbf{r},E)\varphi_p(\mathbf{r},E)=$$

$$\frac{1}{k_p}\sum_m\left[\chi^{(m)}(E)<\nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E')>_{E'}-\sum_i\chi_{d,i}^{(m)}(E)\bar{\nu}_{d,i}^{(m)}<\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E')>_{E'}\right]$$

$$+<\Sigma_s(\mathbf{r},E'\to E)\varphi_p(\mathbf{r},E')>_{E'}.$$

Delayed neutron treatment

The formula $1-(k_p/k)$ corresponds to:

$$\beta_{eff}^{MC}=1-\frac{k_p}{k}=\sum_m\sum_i\frac{<<\varphi^+(\mathbf{r},E)\chi_{d,i}^{(m)}(E)\bar{\nu}_{d,i}^{(m)}<\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E')>_{E'}>_E>_{\mathbf{r}}}{\sum_m<<\varphi^+(\mathbf{r},E)\chi^{(m)}(E)<\nu^{(m)}(E')\Sigma_f^{(m)}(\mathbf{r},E')\varphi_p(\mathbf{r},E')>_{E'}>_E>_{\mathbf{r}}}$$

Check of both members by ERANOS

ERANOS formalism and calculation set up

The following β_{eff} formulation is adopted in ERANOS:

$$\beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}$$

Average value of delayed neutrons (basic data).

Unique delayed neutron spectrum for all the fissile isotopes m, delayed neutron family i (basic data).

$$\bar{\chi}_g = \frac{\sum_m \chi_g^{(m)} \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)} \psi_{g'}}{\sum_m \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)} \psi_{g'}}.$$

$$\mathbf{\Omega} \cdot \nabla \varphi_g(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r}) \varphi_g(\mathbf{r}) = \frac{1}{k} \bar{\chi}_g \sum_m \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) + \sum_{g'} \Sigma_{s,g' \to g}(\mathbf{r}) \varphi_{g'}(\mathbf{r})$$

$$-\mathbf{\Omega} \cdot \nabla \varphi_g^+(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r}) \varphi_g^+(\mathbf{r}) = \frac{1}{k} \sum_m \nu_g^{(m)} \Sigma_{f,g}^{(m)}(\mathbf{r}) \sum_{g'} \bar{\chi}_{g'} \varphi_{g'}^+(\mathbf{r}) + \sum_{g'} \Sigma_{s,g \to g'}(\mathbf{r}) \varphi_{g'}^+(\mathbf{r}).$$

ERANOS formalism and calculation set up

$$\beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \; \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \; \bar{\chi}_g \sum_m \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \; \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}$$

"*improved*" PT first order formulation

$$\beta_{eff}^{MC} = 1 - \frac{k_p}{k} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \; \varphi_{p,g'}(\mathbf{r}) >_{\mathbf{r}}} .$$

$$\boldsymbol{\Omega} \cdot \nabla \varphi_{p,g}(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r}) \varphi_{p,g}(\mathbf{r}) = \frac{1}{k_p} \sum_m \sum_{g'} \left[\bar{\chi}_g \nu_{g'}^{(m)} - \sum_i \bar{\chi}_{d,i,g} \bar{\nu}_{d,i}^{(m)} \right] \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}) + \sum_{g'} \Sigma_{s,g' \to g}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r})$$

?

ERANOS formalism and calculation set up

Simplifications:

1)

$$\bar{\nu}^{(m)} = \frac{\sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)} \psi_{g'}}{\sum_{g'} \Sigma_{f,g'}^{(m)} \psi_{g'}}$$

$$\boldsymbol{\Omega} \cdot \nabla \varphi_g(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r}) \varphi_g(\mathbf{r}) =$$

$$\frac{1}{k} \bar{\chi}_g \sum_m \bar{\nu}^{(m)} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) + \sum_{g'} \Sigma_{s,g' \to g}(\mathbf{r}) \varphi_{g'}(\mathbf{r}),$$

$$-\boldsymbol{\Omega} \cdot \nabla \varphi_g^+(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r}) \varphi_g^+(\mathbf{r}) =$$

$$\frac{1}{k} \sum_m \bar{\nu}^{(m)} \Sigma_{f,g}^{(m)}(\mathbf{r}) \sum_{g'} \bar{\chi}_{g'} \varphi_{g'}^+(\mathbf{r}) + \sum_{g'} \Sigma_{s,g \to g'}(\mathbf{r}) \varphi_{g'}^+(\mathbf{r}),$$

2)

$$\bar{\chi}_{p,g} = \frac{\sum_m \left[\bar{\chi}_g \bar{\nu}^{(m)} - \sum_i \bar{\chi}_{d,i,g} \bar{\nu}_{d,i}^{(m)} \right] \sum_{g'} \Sigma_{f,g'}^{(m)} \psi_{g'}}{\sum_m \left[\bar{\nu}^{(m)} - \sum_i \bar{\nu}_{d,i}^{(m)} \right] \sum_{g'} \Sigma_{f,g'}^{(m)} \psi_{g'}},$$

$$\boldsymbol{\Omega} \cdot \nabla \varphi_{p,g}(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r}) \varphi_{p,g}(\mathbf{r}) = \frac{1}{k_p} \bar{\chi}_{p,g} \sum_m \left[\bar{\nu}^{(m)} - \sum_i \bar{\nu}_{d,i}^{(m)} \right] \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r})$$

$$+ \sum_{g'} \Sigma_{s,g' \to g}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}).$$

ERANOS formalism and calculation set up

The corresponding β_{eff} formulation is:

$$\beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \bar{\nu}^{(m)} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}$$

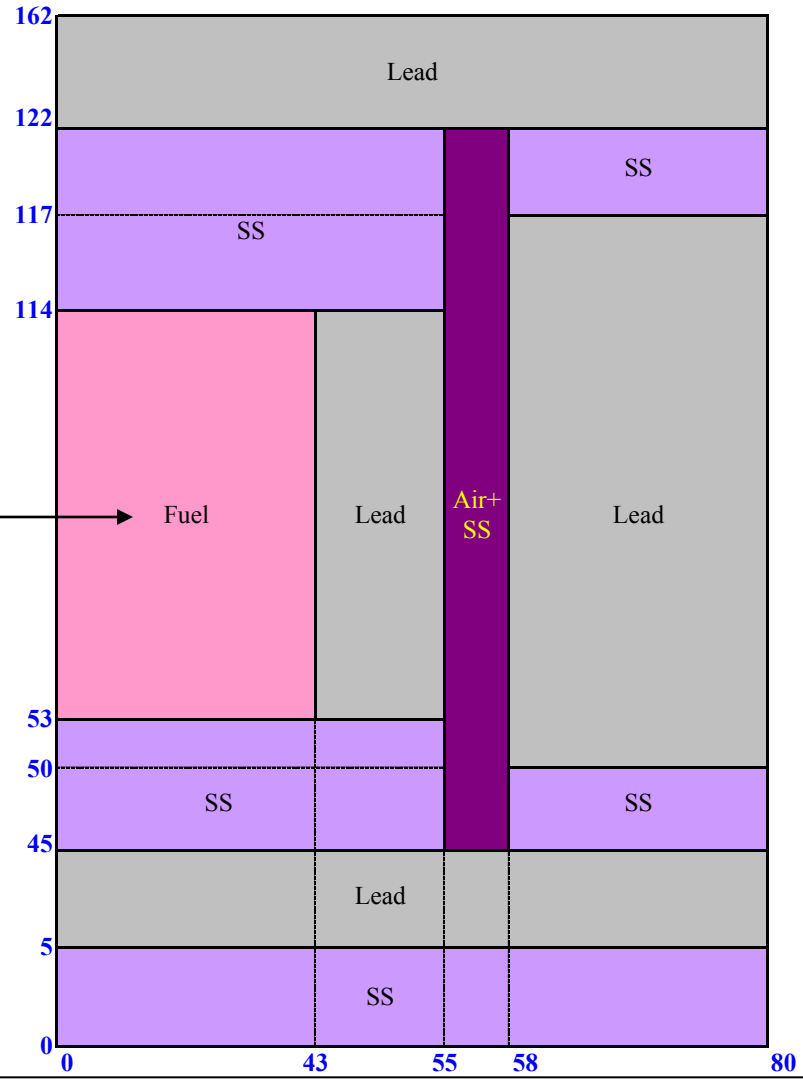
"*improved*" PT first order formulation

$$\beta_{eff}^{MC} = 1 - \frac{k_p}{k} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \bar{\nu}^{(m)} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}) >_{\mathbf{r}}}$$

Now all the equations sides can be calculated by ERANOS

ERANOS results

Simplified RZ model of the GUINEVERE start-up (at critical) configuration. Dimensions, not in scale, are given in cm.



Volumetric fractions:
Stainless Steel 16%,
Lead 60%. Air 7%.
Uranium 17%.

U 30% (weight) enriched in U235

ERANOS results

ERANOS transport calculations

Neutron data library JEFF 3.1 - 49 energy groups - P_0 transport approximation - Angular quadrature S_4

Delayed neutron data

	Family	1	2	3	4	5	6	7	8	Sum
U_{235}	Abundance *	0.0340 (1)	0.1501 (1)	0.0992 (1)	0.2001 (1)	0.3122 (1)	0.0932 (1)	0.0872 (1)	0.0240 (1)	1
	β **	0.00022	0.00099	0.00065	0.00132	0.00205	0.00061	0.00057	0.00016	0.00657
	ν_d	0.00055	0.00245	0.00162	0.00326	0.00509	0.00152	0.00142	0.00039	0.01630 (2)
U_{238}	Abundance	0.0084 (3)	0.1040 (3)	0.0375 (3)	0.1370 (3)	0.2940 (3)	0.1980 (3)	0.1280 (3)	0.0931 (3)	1
	β **	0.00014	0.00175	0.00063	0.00231	0.00496	0.00334	0.00216	0.00157	0.01687
	ν_d	0.00039	0.00484	0.00174	0.00637	0.01367	0.00921	0.00595	0.00433	0.04650 (4)

* Yellow background indicates data from literature. No background indicates derived data.

** $\beta = \bar{\nu}_d / \bar{\nu}$ values are obtained from mean ν values for U_{235} and U_{238} in the GUINEVERE core region.

(1) Note CEA DEN/CAD/DER/SPRC/LEPH 06-204. Data are adapted (to have 1 as summation) from Progress in Nuclear Energy, Vol 41, Number 1-4 2002 (ISSN 0149-1970), pag. 266 (U_{235} fast spectrum).

(2) Progress in Nuclear Energy, Vol 41, Number 1-4 2002 (ISSN 0149-1970), pag. 405 (recommended values U_{235} fast spectrum).

(3) Note CEA DEN/CAD/DER/SPRC/LEPH 06-204. Data are from Progress in Nuclear Energy, Vol 41, Number 1-4 2002 (ISSN 0149-1970), pag. 268 (U_{238} fast spectrum).

(4) Progress in Nuclear Energy, Vol 41, Number 1-4 2002 (ISSN 0149-1970), pag. 405 (recommended values U_{238} fast spectrum).

ERANOS results

$$\beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \sum_{g'} \nu_{g'}^{(m)} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}} \rightarrow \text{723 pcm}$$

$$\beta_{eff} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \bar{\nu}^{(m)} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{g'}(\mathbf{r}) >_{\mathbf{r}}} \rightarrow \text{731 pcm}$$

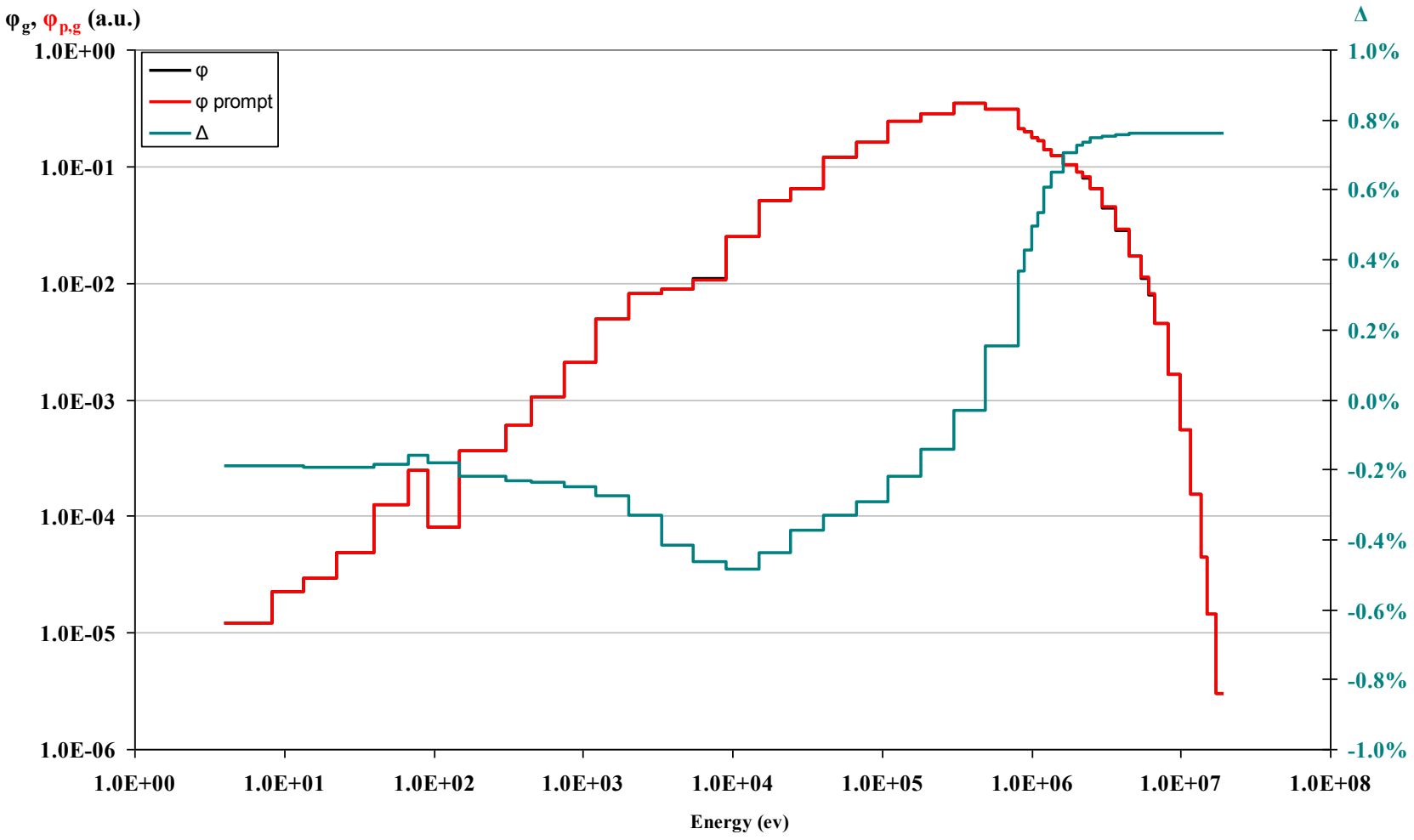
732 pcm

732 pcm

$$\beta_{eff}^{MC} = 1 - \frac{k_p}{k} = \sum_m \sum_i \bar{\nu}_{d,i}^{(m)} \frac{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_{d,i,g} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}) >_{\mathbf{r}}}{< \sum_g \varphi_g^+(\mathbf{r}) \bar{\chi}_g \sum_m \bar{\nu}^{(m)} \sum_{g'} \Sigma_{f,g'}^{(m)}(\mathbf{r}) \varphi_{p,g'}(\mathbf{r}) >_{\mathbf{r}}}$$

ERANOS results

Total and prompt fluxes, together with spectral differences $\Delta = (\varphi_{p,g} - \varphi_g)/\varphi_g$.



Conclusions

- When evaluating β_{eff} by the formula 1-(k_p/k), the *quality* of the obtained result depends on the *quality* of the description of the delayed neutron emission assumed in the k_p prompt calculation.
- Great detail is given in ERANOS to the characteristics of the delayed neutron emission through the β_{eff} formulation, and it is not possible to set up a prompt calculation having the same quality of the delayed neutron emission description available in the β_{eff} formulation.
- Following the perturbation theory approach, a rigorous relationship may be established between the β_{eff} evaluation by the formula 1-(k_p/k), used in Monte Carlo codes, and the corresponding β_{eff} calculation by the classical formulation involving direct and adjoint fluxes.
- The classical formulation can be considered an “improved” PT first order formulation of the formula 1-(k_p/k).
- By means of a refined computational analysis carried out in a coherent and consistent way, i.e. using the same deterministic code ERANOS and neutron data library JEFF 3.1 for the β_{eff} evaluation in different ways, the theoretical analysis is numerically confirmed.

Conclusions (cont'd)

- Both theoretical and numerical results confirm the effectiveness of the β_{eff} evaluation by the MC formula $1-(k_p/k)$, at least in cases where spectral differences between total and prompt fluxes are negligible with respect to the value of the functionals entering the classical β_{eff} formulation.

- For other material configurations more investigations are needed to obtain a detailed quantification of the effects involved.