DE LA RECHERCHE À L'INDUSTRIE

Intermediate Structure in Fission; Consequences on Average Partial Cross Sections

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Collaborative work with LANL-T2

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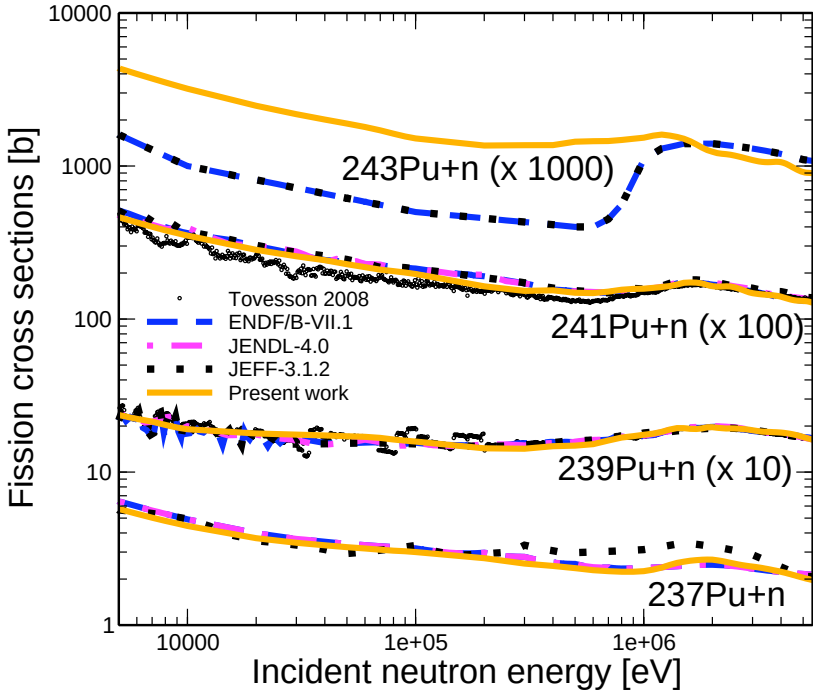
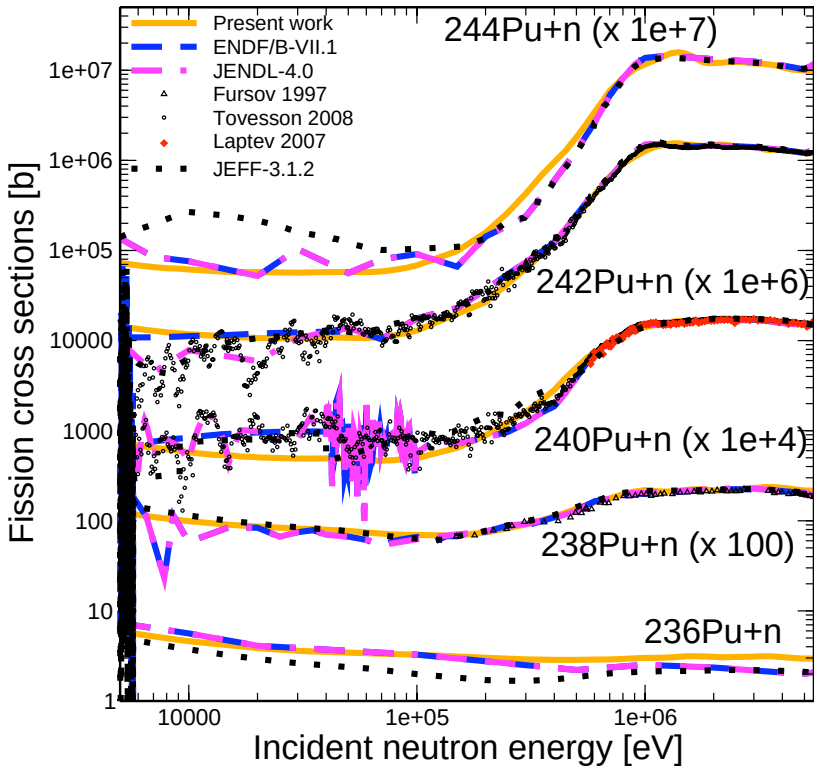
AVERAGE CROSS SECTION SIMULATION: CHALLENGE AND PROBLEMATIC

Heavy fissile nuclei:



Heavy fertile nuclei:
obvious Intermediate Structure (I.S.) effects



How large is the difference in average xs model complexity between fertile and fissile nuclei ?

And so, will the standard HF formalism sufficient for fissile nuclei ?

AVERAGE PARTIAL CROSS SECTIONS
STANDARD HAUSER-FESHBACH FORMALISM

$$\left\langle \frac{\Gamma_c^{J^\pi(ls)}\,\Gamma_{c'}^{J^\pi(l's')}}{\sum_{c''}\Gamma_{c''}^{J^\pi(l''s'')}} \right\rangle = \frac{\bar{\Gamma}_c^{J^\pi(ls)}\,\bar{\Gamma}_{c'}^{J^\pi(l's')}}{\sum_{c''}\bar{\Gamma}_{c''}^{J^\pi(l''s'')}} \times W_{cc'}$$

With

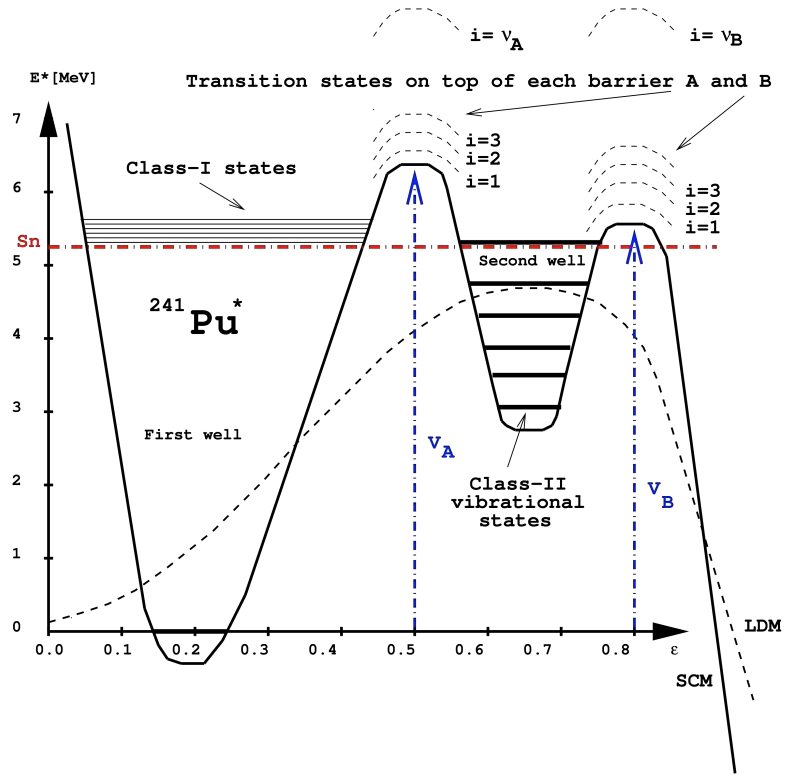
$$T_{c'}^{J^\pi(ls)} \approx 2\pi\frac{\bar{\Gamma}_{c'}}{D_J}$$

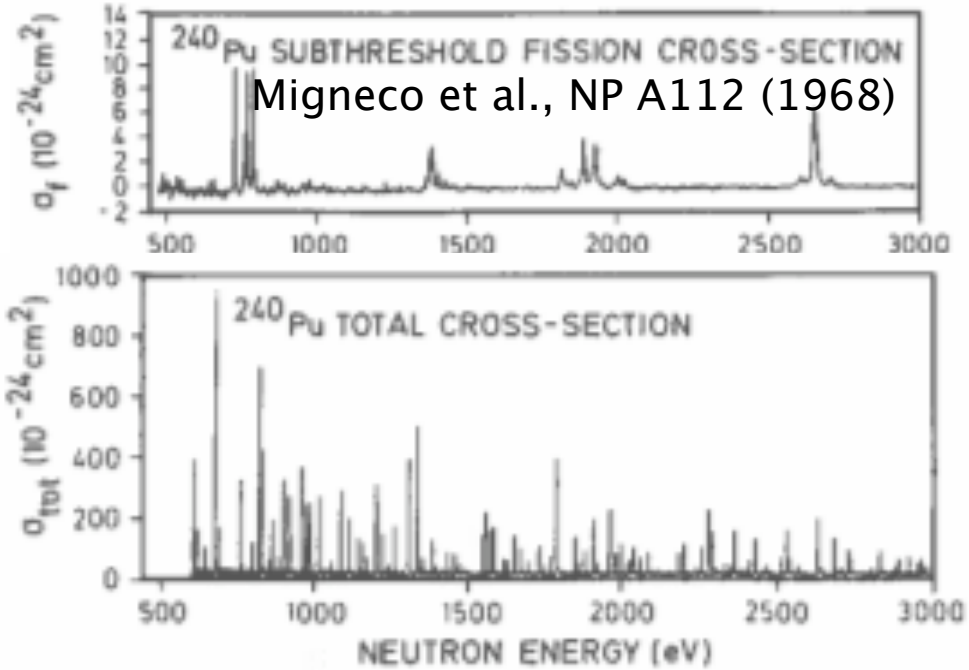
Standard Hauser-Feshbach average cross section with width fluctuation correction factor $W_{cc'}$

$$\bar{\sigma}_{cc'}(E_n) = \sum_J \sigma_c^{J^\pi}(E_n) \sum_{s'=|I'-i'|}^{|I'+i'|} \sum_{l'=|J-s'|}^{|J+s'|} \frac{T_{c'}^{J^\pi(l's')}(E_{c'})}{\sum_{c''} T_{c''}^{J^\pi(l"s")}(E_{c''})} \times W_{cc'}$$

AVERAGE FISSION CROSS SECTION

Strutinsky, Lynn, Weigman have explained the presence of I.S observed by Michaudon, Paya, Migneco (etc.)



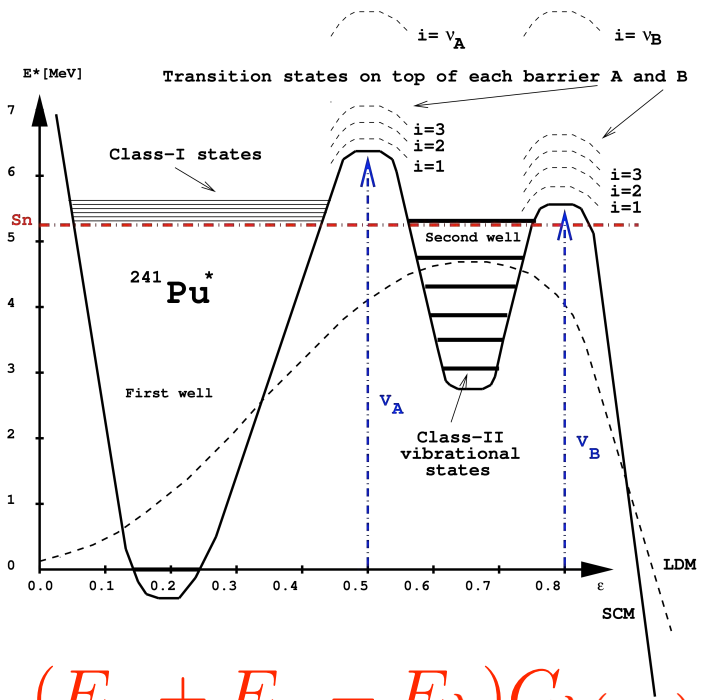


Collective nuclear vibrations and single-particle excited states manifest as I.S effects.

EIGENSTATE CLASSIFICATION: FORMAL R-MATRIX (E. LYNN - 1973)

$$H = H_\eta + H_{int}(\zeta, \eta_0) + H_c(\eta, \zeta; \eta_0) \longrightarrow H X_\lambda = E_\lambda X_\lambda$$

$$\Phi_\nu, E_\nu \qquad \chi_\mu, E_\mu$$



$$X_{\lambda_i} = \sum_{\nu'\mu'} C_{\lambda(\nu'\mu')} \Phi_{\nu'} \chi_{\mu'}$$

Class-I or class-II classification is depending on the location of the largest vibrational amplitude.

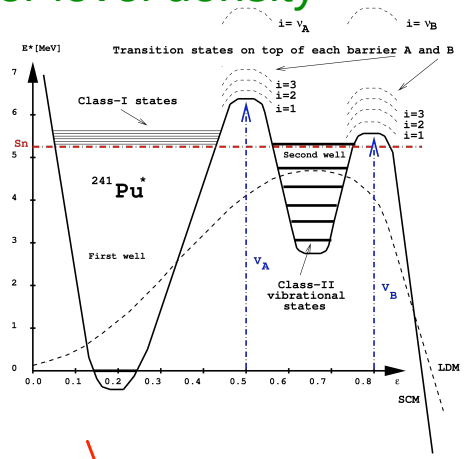
$$(E_\nu + E_\mu - E_\lambda) C_{\lambda(\nu\mu)} + \sum_{\nu'\mu'} C_{\lambda(\nu'\mu')} \langle \Phi_\nu \chi_\mu | H_c | \Phi_{\nu'} \chi_{\mu'} \rangle = 0$$

CLASS-I AND CLASS-II PROPERTIES

λ_I: by nature, class-I have very small fission widths and high level density

$$\Gamma_{\lambda_I,tot} \approx \Gamma_{\lambda_I,n} + \Gamma_{\lambda_I,n'} + \Gamma_{\lambda_I,\gamma_I}$$

λ_{II} : whereas class-II have much larger fission widths and lower level density at the same excitation energy



$$\Gamma_{\lambda_{II},tot} \approx \Gamma_{\lambda_{II\downarrow}} + \Gamma_{\lambda_{II\uparrow}} + \Gamma_{\lambda_{II},\gamma_{II}}$$

We define

$$T_A = 2\pi \frac{\langle \Gamma_{\lambda_{II\downarrow}} \rangle_{II}}{D_{II}} \quad \text{and} \quad T_B = 2\pi \frac{\langle \Gamma_{\lambda_{II\uparrow}} \rangle_{II}}{D_{II}}$$

with $\Gamma_{\lambda_{II\downarrow}} = 2\pi \langle \langle \lambda_{II} | H_c | \lambda_I \rangle^2 \rangle_I / D_I$

CLASS-I CLASS-II COUPLING COMMON SITUATIONS

$$\bar{\sigma}_{cf}^{J^\pi}(E_n) = \sum_J \sigma_c^{J^\pi}(E_n) \times \bar{P}_f \times W_{cf}$$

$$= \sum_J \sigma_c^{J^\pi}(E_n) \sum_{s'=|I'-i'|}^{|I'+i'|} \sum_{l'=|J-s'|}^{|J+s'|} \frac{T_f^{J^{\pi(l's')}}(E_{c'})}{\sum_{c''} T_{c''}^{J^{\pi(l''s'')}}(E_{c''})} \times W_{cf}$$

Statistical approximation (strong damping) – across an outer Bohr channel μ

$$T_f(\mu) = \frac{T_A T_B(\mu)}{T_A + T_B}$$ and $$T_f = \sum_\mu T_f(\mu)$$

Moderately weak coupling: average sub-barrier penetrability
(Lynn and Back formula – 1974)

$$\bar{P}_f = \left[1 + \left(\frac{T_{I,tot}}{T_f}\right)^2 + \left(\frac{2T_{I,tot}}{T_f}\right) coth\left(\frac{T_A + T_B}{2}\right)\right]^{-1/2}$$

text

COUPLING AND FISSION WIDTH CORRELATIONS: W_{II} FACTOR

Thus we have to consider the overall fission transmission across the double barrier (depending on the damping strength in the second well)

$$\langle T_f \rangle(\mu) \approx \left\langle \frac{T_A T_B(\mu)}{T_A + T_B} \right\rangle_{\lambda_{II}} = \frac{2\pi}{D_{II}} \left\langle \frac{\Gamma_{\lambda_{II}\downarrow} \Gamma_{\lambda_{II}\uparrow}(\mu)}{\Gamma_{\lambda_{II}}} \right\rangle_{\lambda_{II}}$$

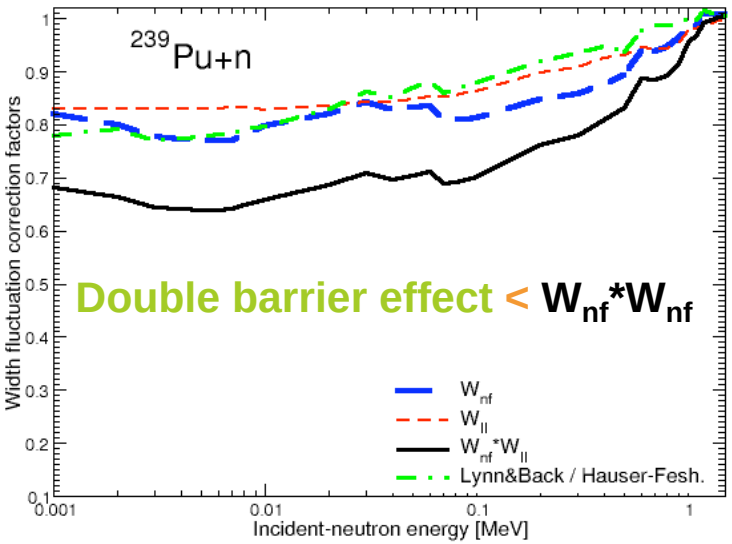
Possible width correlations will perturb the P&T distribution of the final R-matrix eigenstate fission widths.

$$\left\langle \frac{\Gamma_{\lambda_{II}\downarrow} \Gamma_{\lambda_{II}\uparrow}}{\Gamma_{\lambda_{II}}} \right\rangle_{\lambda_{II}} = \sum_{\mu} W_{II}(\mu) \frac{\bar{\Gamma}_{\lambda_{II}\downarrow} \bar{\Gamma}_{\lambda_{II}\uparrow}(\mu)}{\bar{\Gamma}_{\lambda_{II}}}$$

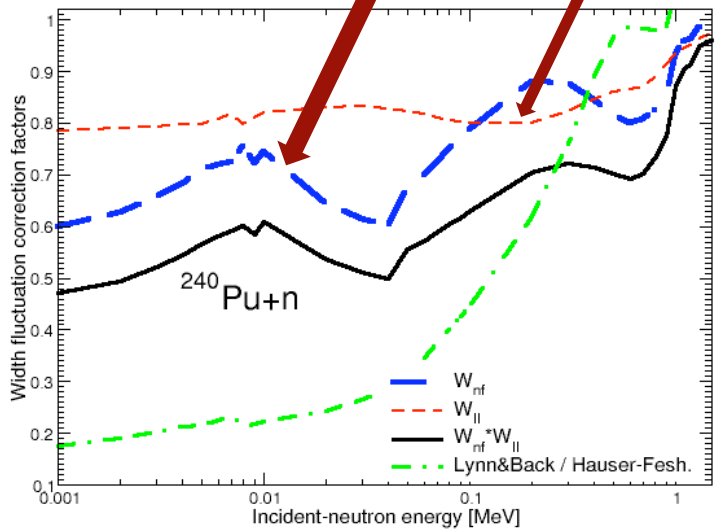
FORMAL R-MATRIX AVERAGE FISSION XS

$$\bar{\sigma}_{nf}^{J^\pi}(E_n) = \sum_J \sigma_n^{J^\pi}(E_n) \sum_{s'=|I'-i'|}^{|I'+i'|} \sum_{l'=|J-s'|}^{|J+s'|} \frac{T_f^{J^{\pi(l's')}}(E_{c'})}{\sum_{c''} T_{c''}^{J^{\pi(l''s'')}}(E_{c''})} \times W_{nf} \times W_{II}$$

Double barrier effect >> $W_{nf}*W_{nf}$



Energy [keV]	1.	100.
W_{nf}	0.80	0.84
W_{II}	0.86	0.88
$W_{nf} \times W_{II}$	0.70	0.74



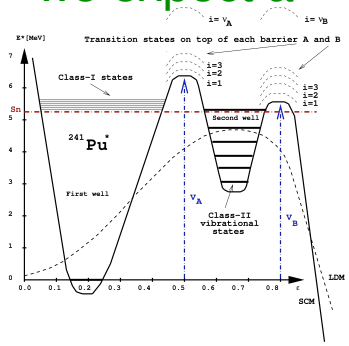
Energy [keV]	1.	100.
W_{nf}	0.60	0.76
W_{II}	0.78	0.78
$W_{nf} \times W_{II}$	0.47	0.59

ν EFFECTIVE COLLATERAL DAMAGE ON THE STANDARD FLUCTUATION FACTOR W_{nf}

Explicit I.S. effect with Lorentzian approximation (Weak coupling)

$$\Gamma^{1/2}_{\lambda_i(\lambda_{I_i}),f} = \sum_{\lambda_{II}} \sum_{\alpha} \langle \lambda_i | \lambda_{I_i} \rangle \langle \lambda_{I_i} | \alpha \rangle \frac{\langle \alpha | H_c | \lambda_{II} \rangle \Gamma^{1/2}_{\lambda_{II}\uparrow}}{(E_{\lambda_i} - E_{\lambda_{II}}) + i\Gamma_{\lambda_{II}}/2}$$

In case of single hump across a single effective outer barrier channel -- > we expect a Porter-Thomas distribution of level fission widths (ν_{eff} = 1) ,



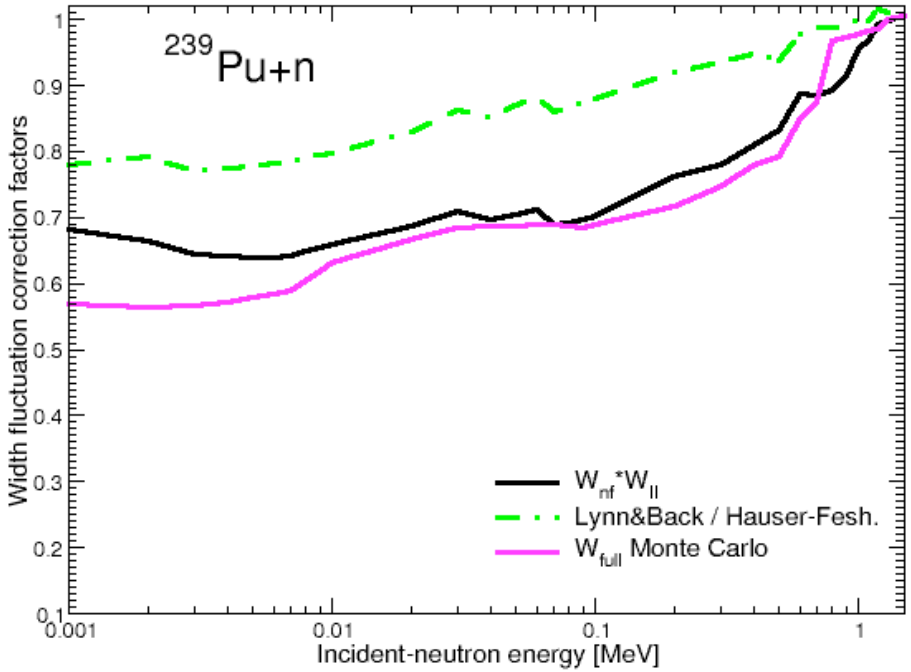
In the case of an explicit I.S calculation and defining

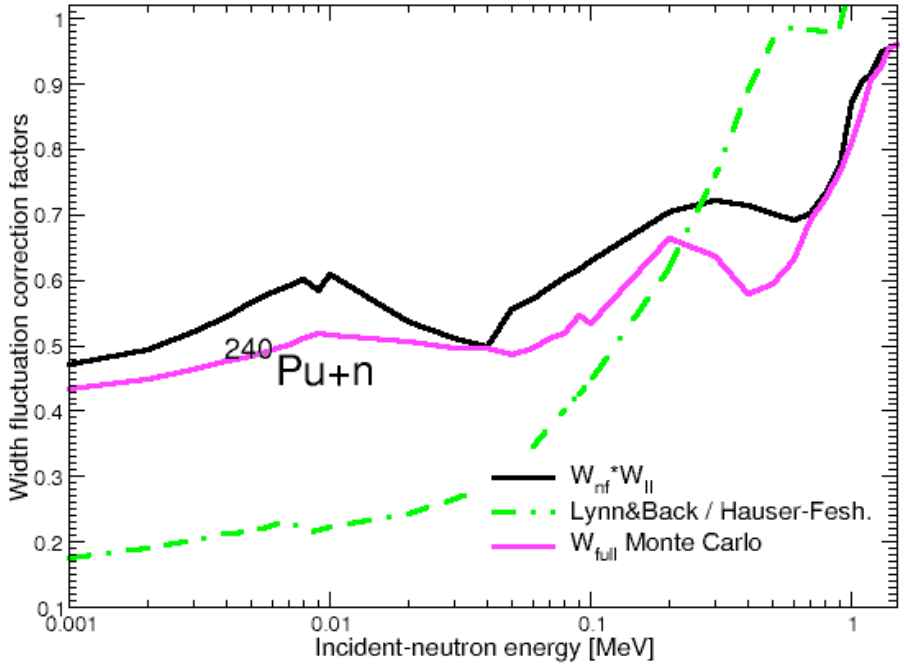
$$\nu_{eff} = \frac{\left(\sum_\alpha \langle \Gamma_{\lambda_f}(\alpha) \rangle\right)^2}{\sum_\alpha \langle \Gamma_{\lambda_f}(\alpha) \rangle^2}$$

ν_{eff}	$^{240*}\mathrm{Pu}\ (J^\pi = 0^+)$	$^{241*}\mathrm{Pu}\ (J^\pi = 1/2^+)$
1 keV	0.72	0.66
100 keV	0.74	0.66

Reduced ν_{eff} values are requested for correct W_{nf} calculations

NESTED DOUBLE BARRIER MONTE CARLO FISSION CROSS SECTION CALCULATIONS





Nested Monte Carlo-type calculations are a powerful alternative to analytical decoupled expressions of sub-barrier and fluctuation effects,

The resulting MC fission cross sections are even lower in magnitude than those calculated from analytical formulae.

CONCLUSIONS

1. Proof is made of the existence of a W_{II} factor (>15% below 100keV),

2. The correlation between coupling ($\Gamma_{\downarrow}$) and fission ($\Gamma_{\uparrow}$) class-II widths reduces significantly the DoF of the final R-matrix eigenstate fission width (Γ_f) distribution

Porter-Thomas hypothesis is invalid

3. Sub-barrier and fluctuation effects are strongly nested and Monte Carlo type calculations are definitively the alternative to common (or exotic) analytical formulae.

To the question raised as preamble,

Can we calculate average fission cross section from standard HF formulation with only entrance-outgoing channel fluctuation factor?

The answer is "**NOT EVEN TRUE FOR FISSILE ISOTOPES**" with an expected error of 20 % on cross section magnitude below 500 keV.

Thanks for your attention_

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