THE NUCLEAR OPTICAL MODEL
INTRODUCTORY OVERVIEW

P.E. Hodgson Nuclear Physics Laboratory,Department of Physics,University of Oxford, Oxford, UK

Abstract

The formalism of the optical model is briefly reviewed, together with the ways it can be used to analyse experimental data. The usefulness and limitations of global potentials is assessed. The justification for using optical potentials to give distorted waves for nuclear reaction calculations is examined and some remaining problems discussed. The final section is devoted to the application of optical model potentials to calculate reaction cross-sections at energies up to 200 MeV.

1. Introduction

During the last forty years the nuclear optical model has been extensively applied to analyse the elastic scattering of pions, nucleons and heavier particles by nuclei over a wide range of energies [1]. It has been extended to include inelastic scattering by the coupled-channels formalism [2] and consideration of dispersion effects enables both bound and scattering states to be described by the same mean field [3].

The interaction of a nucleon with a nucleus is inherently complicated, and the optical model represents it by a phenomenological potential of very simple form, with parameters that are adjusted to fit the experimental data. This potential has an imaginary part that takes into account the absorption of the reaction flux from the elastic channel to the non-elastic reaction channels. This is analogous to the scattering and absorption of light by a medium of complex refractive index, which is why it is called the optical model.

Elastic scattering cross-sections are affected by the excited states of the compound system and by the residual states in the non-elastic channels. This causes the cross-section to fluctuate, and the fluctuations may be seen and analysed when measurements of high precision are made on light nuclei. Usually the experimental resolution of the detector is sufficient to average over these fluctuations, but it must be emphasised that the optical model applies only to energy-averaged cross-sections.

The optical model is sometimes criticised for its many parameters: with so many you could fit anything! However the flexibility of the model is limited, and the form and the magnitude of the real part of the potential, and to a lesser extent the imaginary part, can be estimated quite well from simple physical considerations. Furthermore, the strength of its physical basis is confirmed by its ability to fit with very similar parameters the scattering from many nuclei over a large energy range including the bound states of the simple shell model. To achieve this, the depths of the potential vary smoothly with energy, and this can be attributed in the case of the real part of the potential to its non-local nature.

Optical potentials obtained by analysis of elastic scattering are widely used to generate the distorted waves used to analyse the cross-sections of many reactions, and these analyses have proved to be a powerful tool in determining nuclear structures.

In this review, the forms of the optical potential are described in Section 2, and the methods of optical model analysis in Section 3. In spite of its many successes, the optical model is still being developed and there remain some problems to study. Among these are the validity of the distorted waves generated by optical potentials and the usefulness of reaction cross-sections, and this is discussed in Section 4. The concluding Section 5 is devoted to reactions up to 200 MeV.

2. Optical Model Potentials

Over the years optical model potentials have become increasingly sophisticated to enable them to account for more accurate and extensive experimental data.

The early potentials had the form

$$V = (U + iW)f(r) \tag{2.1}$$

where U and W are the real and imaginary potential depths and $f(r)$ is the radial form factor. This usually has the Saxon-Woods form

$$f(r) = \frac{1}{1 + \exp\left(\frac{r-R}{a}\right)} \tag{2.2}$$

where R and a are the radius and surface diffuseness parameters. This is the four-parameter optical potential. When inserted into the quantum mechanical formalism it gives the differential cross-section for elastic scattering and reaction (absorption) cross-sections for charged particles, and for neutral particles the total elastic cross-section and the total cross-section as well.

Analyses of more accurate data showed that a better fit could be obtained by allowing the form factor parameters to differ for the real and imaginary potentials; this gives the six-parameter potential.

At low energies the interaction takes place predominantly in the surface region of the nucleus, so some analyses use a surface-peaked form factor for the imaginary part of the potential having the form

$$g(r) = -4a\frac{d\,f(r)}{dr} = \frac{4\exp[(r-R)/a]}{\{1+\exp[(r-R)/a]\}^2} \tag{2.3}$$

where the factor $-4a$ is introduced so that $g(R) = 1$. The imaginary potentials with $f(r)$ and $g(r)$ are referred to as volume and surface absorption respectively. It is possible to include both surface and volume absorption terms and this is necessary for the dispersion relation analyses described below. Since the cross-sections are rather insensitive to the form of the potential, very accurate data are necessary for these analyses. In some early analyses a Gaussian form was used for $g(r)$.

The addition of a spin-orbit term enables the polarisation of the scattered particle to be calculated. This term has the form

$$V_{so}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 (U_s + iW_s)\frac{1}{r}\frac{d\,f_s(r)}{dr}\mathbf{L}.\boldsymbol{\sigma} \tag{2.4}$$

The square of the pion Compton wavelength is a relic of the derivation of this term from the meson theory of nuclear forces, and its numerical value is close to 2 fm^2. The form factor $f_s(r)$ has the Saxon-Woods form (2.2) and usually a rather smaller value of the radius parameter. There is very little evidence favouring the inclusion of the imaginary spin-orbit potential, so it is usually omitted. This gives the nine-parameter optical potential.

Since the optical potential represents all the nucleon-nucleon interactions between the incident particle and the target nucleus it is possible to estimate nucleon optical potentials by summing the nucleon-nucleon interactions over the nuclear density distribution $\rho(r)$, giving

$$V(r) = \sum_{i=1}^{A} v(|\mathbf{r} - \mathbf{r}_i|) \approx \int \rho(r')v(|\mathbf{r} - \mathbf{r}'|)d\mathbf{r}' \tag{2.5}$$

where $v(|\mathbf{r} - \mathbf{r}'|)$ is the effective interaction between the incident nucleon and a nucleon in the nucleus. This effective interaction can be represented by a Gaussian or a Yukawa form, with adjustable range and strength. This folded potential is useful only for the real part of the potential, so when it is used it is supplemented by a phenomenological imaginary part as before. Folded potentials are more frequently used for alpha-particle and heavy ion scattering, since they often give a better representation of the surface part of the potential than the phenomenological forms (2.1) to (2.4). An additional refinement is the inclusion of a density-dependent term in the expression for the effective interaction. Double folded potentials can similarly be obtained for composite projectiles.

These optical potentials treat explicitly only the elastic scattering channel, and all other non-elastic channels are taken into account in a global way by the imaginary potential that represents the total flux

removed from the elastic channel by all other reactions. This is referred to as the simple optical model (SOM).

It is also possible to treat explicitly one or more inelastic scattering channels by the coupled-channels formalism [2]. This consists of a set of coupled equations that can be solved to give both the elastic and the inelastic scattering in as many channels as required. In addition to the optical potential, such calculations also require a model of the nuclear excitation, so that the coupling terms can be evaluated. A coupled-channels analysis is made when there is strong coupling between the elastic channel and the inelastic channels that lead to the excitation of low-lying collective states. Since the computing time increases approximately as the cube of $\sum_J (2J+1)$, where J is the total angular momentum of the target states, it is practicable to couple only a small number of channels. The inelastic scattering with excitation of the higher states can be found using the distorted wave Born approximation.

The optical potential can be regarded as the positive energy part of the mean field that is experienced by a nucleon in the region of a nucleus. The negative part is the potential of the simple shell model that gives the allowed bound states and hence the magic numbers. Examination of this mean field over the whole energy range shows that it is essentially continuous as a function of energy. The real part decreases monotonically with increasing energy, and the imaginary part has a parabolic variation around the Fermi surface energy. For negative energies, the imaginary potential is connected to the fragmentation width of the bound states.

In the region of the Fermi surface the real potential depth deviates from the regular behaviour due to dispersion effects. The real and imaginary pats of the potential are connected by the dispersion relations that follow from the requirement of causality namely that the scattering wave is not emitted before the incident wave arrives [4]. Thus, since the optical potential $V(K,E)$ is a complex analytic function

$$V(K,E) = \frac{1}{2\pi i} \oint_c \frac{V(K,E')}{E'-E} dE' \tag{2.6}$$

where the contour encloses the singularity at $E = E'$.

Putting

$$V(K,E) = U(K,E) + iW(K,E) \tag{2.7}$$

and separating into real and imaginary parts gives the dispersion relations

$$U(K,E) = \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} \frac{W(K,E')}{E'-E} dE' \tag{2.8}$$

and

$$W(K,E) = -\frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} \frac{U(K,E')}{E'-E} dE' \tag{2.9}$$

where $\mathcal{P}$ indicates the principal part of the integral.

These dispersion relations further unify the optical potential by relating the real and imaginary parts, but there is the practical difficulty that the potentials are required over the whole energy range from minus to plus infinity. Several methods have been developed to overcome this difficulty. Using the symmetric behaviour of the imaginary potential about the Fermi surface gives

$$U(E,r) = U_{HF}(E,r) + \frac{2}{\pi}(E-E_F) \int_{E_F}^{\infty} \frac{W(E',r)}{(E'-E_F)^2 + (E-E_F)^2} dE' \tag{2.10}$$

where E_F is the Fermi energy. This integral has satisfactory convergence. The Hartree-Fock potential $U_{HF}(E,r)$ has a volume form and falls smoothly and almost linearly with increasing energy. The imaginary potential $W(E',r)$ has both volume and surface parts; the volume part is absorbed into the Hartree-Fock term, while the surface part modulates the potential around the Fermi energy. To illustrate this, the energy variations of the volume (W_V) and surface (W_D) imaginary potentials for neutrons on ^{40}Ca, together with the corresponding dispersive additions to the real potential, are shown in Fig.1 [5]. The energy dependence of the volume integrals of the real potential for neutrons and protons on ^{208}Pb are shown in Fig.2 [6]. It is apparent that the modulation due to the dispersion effect is appreciable only at low energies. It is, however, essential to include it in this energy region, as shown in Fig.3 [7].

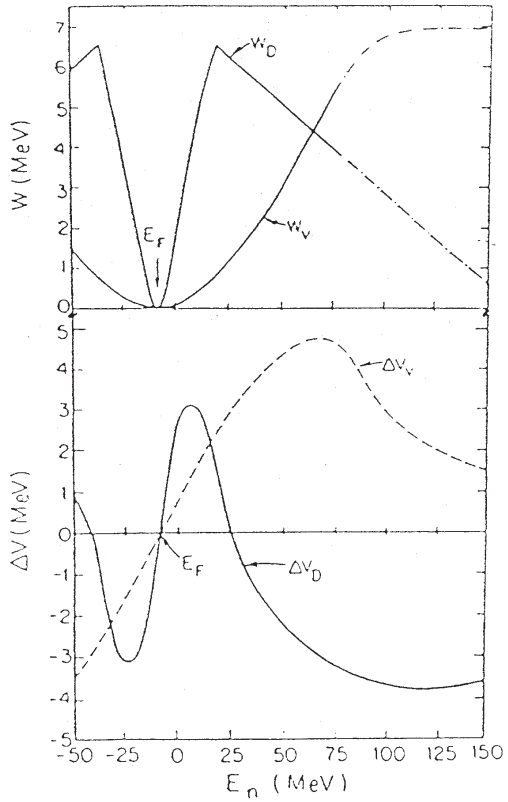

Fig. 1. Energy variations of the volume (W_V) and surface (W_D) imaginary potentials for neutrons on ^{40}Ca, together with the corresponding dispersive additions to the real potential (Delaroche and Tornow, [5]).

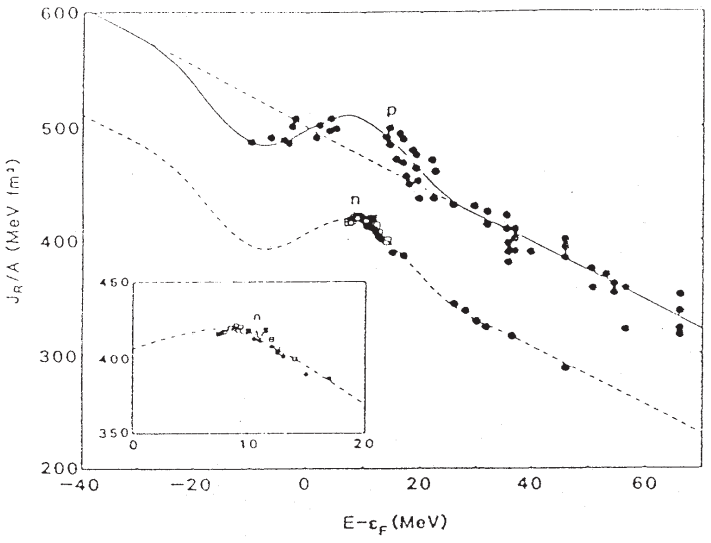

Fig. 2. Energy dependence of the volume integral of the real potential for neutrons and protons on ^{208}Pb showing the Fermi surface anomaly (Finlay *et al*, [6]).

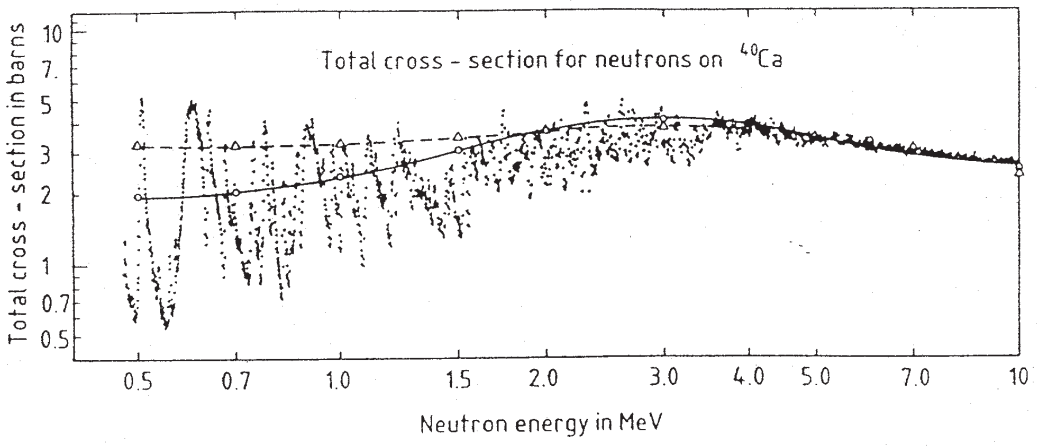

Fig. 3. Total cross-section for the interaction of neutrons with ^{48}Ca compared with calculations using a global optical potential with (full line) and without (dashed line) the dispersion term [7].

3. Optical Model Analyses

In order to use the optical model to calculate the differential cross-section for elastic scattering, and if necessary also the polarisation, it is necessary to specify the potentials numerically. In the case of nucleon scattering, approximate values of the parameters of the potential can be obtained from elementary physical considerations. The depth of the real potential is similar to that of the simple shell model, which is known from studies of bound single-particle states to be about 50 MeV. The splitting of these states gives 4 MeV for the spin-orbit potential. The depth of the imaginary part of the potential can be estimated semi-classically from the total absorption cross-section to be about 8 MeV. The form factor parameters can be obtained from the folding model potential (2.5). Since the effective nucleon-nucleon interaction is short range, it can be approximated by a delta function, and then (2.5) becomes

$$V(r) \propto \rho(r). \tag{3.1}$$

The nuclear density $\rho(r)$ can be represented quite well by the Saxon-Woods form (2.2), with $R = 1.25A^{1/3}$ and $a = 0.6$fm.

To analyse a set of experimental data, which is usually a differential elastic scattering cross-section and possibly also polarisations, a reaction cross-section and (for neutrons) a total cross-section, these quantities are calculated from a potential with initial parameter values such as the above, and then compared with the data. An automatic search routine then varies the parameters systematically until the optimum fit is obtained. Many analyses have been made in this way, and the resulting potentials are available in the literature or in tabular form [8], so quite accurate starting values can be readily obtained.

One of the great strengths of the optical model is that it is possible to fit the elastic scattering by many nuclei over a range of energies with very similar parameters. This makes it possible to obtain global potentials that are very useful in reaction analysis, since we often require distorted waves for energies and target nuclei for which no elastic scattering measurements are available.

To obtain a global potential it is usual to analyse many sets of elastic scattering data, using form factor parameter fixed to average values determined from preliminary analyses where they were allowed to vary. The resulting potential depths usually have a smooth variation with energy and from nucleus to nucleus, and these can be parameterised. The dependence on nuclear asymmetry can be included for nucleons. Since global potentials are seldom required at low energies it is usually not necessary to include dispersion effects, but these can be included if desired.

Many global parameter sets have been obtained in this way, and in order to select the best it is very useful to make comparisons with a wide range of data, as has been done for neutrons by Young [9].

Global potentials take no account of the very different structures of particular nuclei, and so we cannot expect them to fit the data perfectly; indeed it is rather surprising that they usually fit as well as

they do. It is always possible to improve the fit substantially to any particular set of data by varying the parameters, and such potentials may be better than global ones in reaction analyses. There are however serious problems with such analyses due to ambiguities that are discussed in section 4.

The optical potential is applicable when the cross-sections are energy-averaged to remove the fluctuations due to individual states in the compound system. It therefore cannot be applied at low energies. At high energies relativistic effects must be included. Global optical potentials thus apply over a limited energy range.

There are quite satisfactory global potentials for neutrons [9] and protons [10], and some for deuterons [11]. Despite sustained efforts there are none for helions and alpha-particles. This may be due to the greater sensitivity of the scattering to nuclear structure, particularly in the surface region.

An essential preliminary to any optical model analysis is to verify that the computer program is correct. This may be done by comparing some trial calculations with the results published in program intercomparisons [12].

4. The Use of Optical Model Potentials in Reaction Analyses

Most quantum-mechanical analyses of nuclear reaction data use distorted waves generated by optical potentials fitted to the appropriate elastic scattering data or obtained from a global parametrisation. It is however possible to find many optical potentials that fit the elastic scattering data equally well, and these frequently give substantially different distorted waves and hence different nuclear reaction cross-sections. This is a serious problem that deserves more attention than it has so far received.

The source of the difficulty is that the optical potential describes with rather few parameters the elastic scattering data that is exactly specified by a much larger number of S-matrix elements. It is adjusted to give the best overall fit to elastic scattering, but it may not be the best for reaction analyses. It may well be that the elastic scattering is more sensitive to one set of matrix elements and the reaction to another set.

One way to overcome this difficulty is to require the optical potential to fit selected reaction data as well as elastic scattering data. Another possibility is to extract the S-matrix elements from the experimental data using inversion techniques, but these are not without their difficulties.

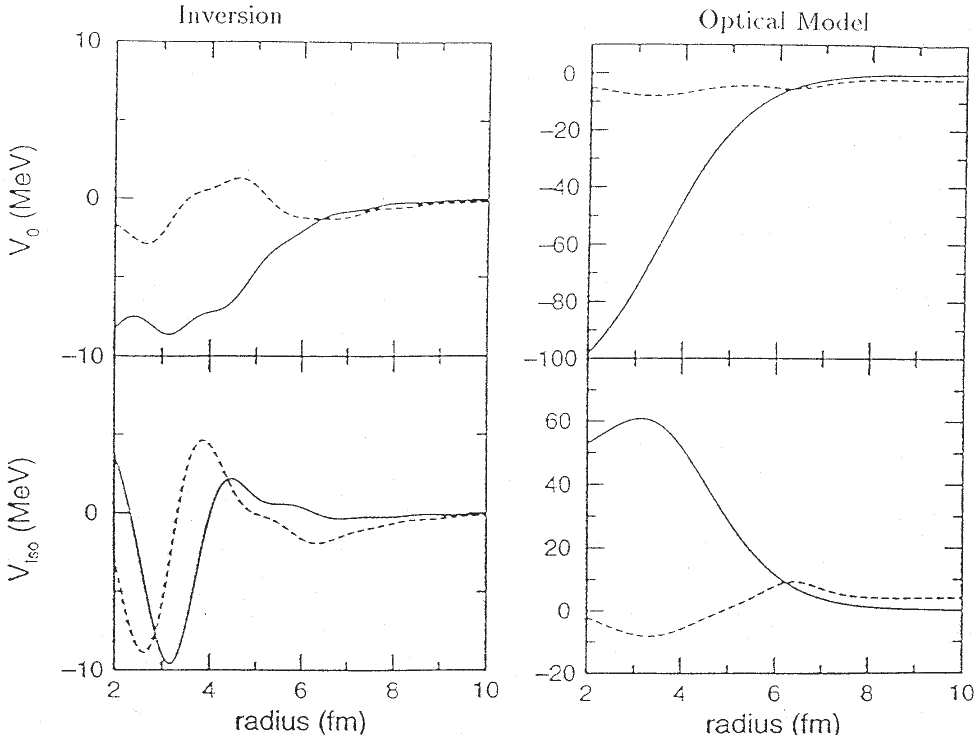

Fig. 4. The potentials obtained by inversion and by optical model analysis of the elastic scattering of 334 MeV tritons and 72 MeV ^{3}He by ^{14}C. The full lines show the real parts and the dashed lines the imaginary parts of the central potential V_0 and the isospin potential V_{iso} [16].

As an example, Fig. 4 shows two quite different potentials obtained by optical model analysis and by inversion that both give excellent fits to the elastic scattering of 334 MeV tritons by ^{14}C and 72 MeV, ^{3}He by ^{14}C.

Another difficulty concerning the distorted waves is that the optical model is determined by fitting the asymptotic values of the partial waves whereas the cross-section of a reaction depends on the wavefunction throughout the nucleus. In addition the effect of the non-locality of the potential should be considered.

Since the reaction cross-sections are the sum of the cross-sections in all the reaction channels, they may provide useful additional information. Many measurements of reaction cross-sections have been made, and the results usually agree reasonably well with those calculated from optical potentials fitted to the elastic scattering cross-sections. Reaction cross-sections are however rather difficult to measure, and until recently the values obtained were not accurate enough to restrict the choice of optical potential.

Recent work has however given more accurate values that may enable this to be done. Thus Auce *et al* [13] have measured the cross-sections of the scattering of 75–190 MeV alpha-particles by nuclei from ^{12}C to ^{208}Pb. The experimental reaction cross-sections were found to be 10–20% less than those calculated from optical potentials fitted to the elastic scattering cross-sections. This also could be attributed to different sensitivities to different sets of S-matrix elements. However there is apparently some difficulty with these measurements that has yet to be resolved.

Very recently, some accurate measurements by Yamaya *et al* [14] of the forward glory scattering of ^{12}C, ^{13}C, ^{15}N and ^{16}O by ^{28}Si at energies about twice the Coulomb barrier have been analysed to give values of the reaction cross-section that are more accurate than those obtained from optical model analyses, but consistent with them.

An ingenious method to extract reaction cross-sections from elastic scattering data has been proposed by Masaki and Aoki [15], but this has not yet been sufficiently tested.

It may well be that the only way to obtain potentials free from these ambiguities is to calculate them from the nucleon-nucleon potential. As an example of such a calculation, Karantaghitic *et al* [16] have used the Paris potential to calculate the G-matrix elements and hence, using the local density approximation, the differential cross-sections and analysing powers. These calculations, however, are inherently complicated, and the potentials obtained are very non-local. It is thus not possible to relate them to the corresponding optical potentials.

5. Reactions at energies up to 200 MeV

The optical model can be used up to about 200 MeV without appreciable difficulties due to relativistic effects. There is some evidence that the form factors used at lower energies are no longer adequate, and there are few global potentials that extend to such high energies.

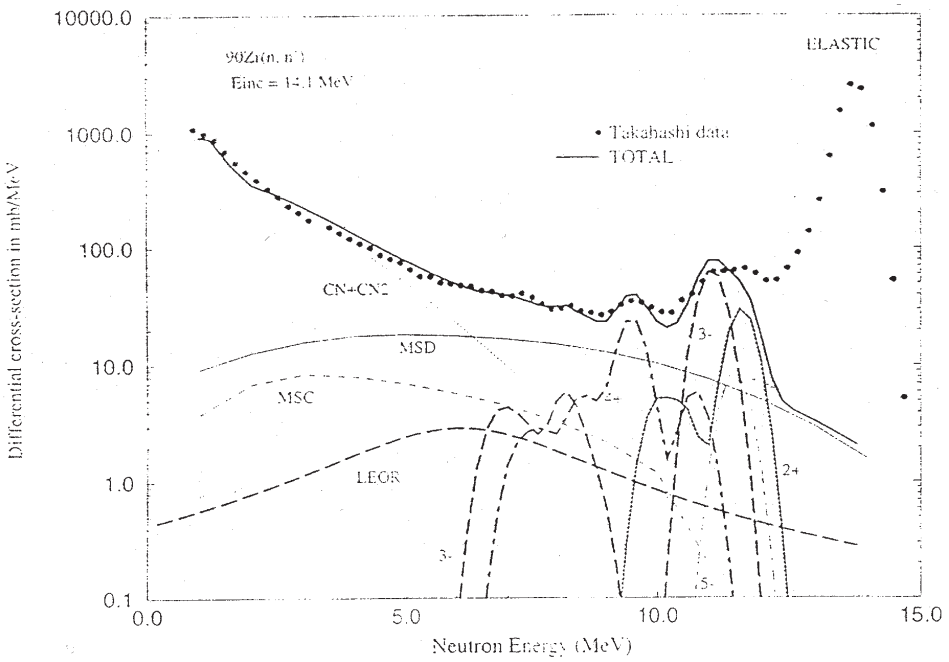

Fig. 5. The energy spectrum of neutrons inelastically scattered from ^{90}Zr at 14.1 MeV compared with MSD (thin solid curve), MSC (thin dashed curve), CN (thin dotted curve), secondary compound nucleon emission (CN2) (thin long-dashed curve) and collective cross-sections calculated as described in the text. The collective excitations contribute to the continuum and include the 2^+ (thick dotted curve), 3^- (thick long-dashed curve), 4^+ (thick dot-dashed curve, 5^- (thin dot-dashed curve) states and the LEOR (thick long dashed curve). The thick full curve is the incoherent sum of all these cross-sections [17].

In many nuclear reactions there are many reaction channels and multistep processes contribute strongly to the measured cross-sections. As an illustration of this complexity, Fig.5 shows the angle-integrated cross-section for the inelastic scattering of 14 MeV neutrons by ^{93}Nb. At the higher outgoing energies, corresponding to the lower excitation energies, there are resolved peaks corresponding to the low-lying collective states; these cross-sections may also be calculated using the coupled-channels formalism. At the lower outgoing energies there are contributions from the higher multipole resonances, including the octupole resonance, and from the multistep compound [18] and multistep direct [19] reactions. Finally at the lowest outgoing energies there are the compound nucleus reactions. It used to be thought that reactions could take place by just two processes, direct and compound nucleus, and that the experimental

data could be reproduced by evaluating these separately and then adding them incoherently. In some cases this is quite a good approximation, but in others it is seriously inadequate.

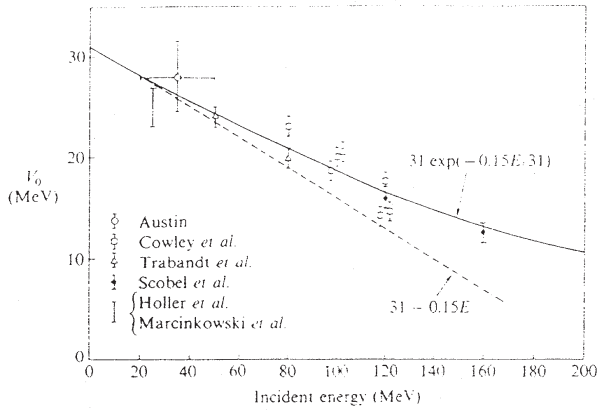

Fig. 6. The effective interaction strength V_0 as a function of incident energy for nucleons with the Yukawa form factor with range 1 fm. The line $V_0 = 31\exp(-0.15E/31)$ is obtained from the energy dependence of the optical model [20].

As the incident energy increases, the contributions of the compound nucleus, multistep compound and multipole resonances fall, until by about 50 MeV only the multistep direct remains, except perhaps at small outgoing energies. The cross-sections for these reactions may be calculated using the multistep direct theory with standard optical potentials. The only parameter in this theory is the effective interaction strength V_0 which may be derived from the folding model expression for the real part of the optical potentials. A comparison between values of V_0 calculated in this way with those obtained by analysing experimental (p, p') cross-sections is given in Fig.6. Many double-differential (p, p') cross-sections have been well fitted with the multistep direct theory [20], and recently it has been extended to analyse (p, α) and $(p, {}^3\mathrm{He})$ cross-sections from 20 to 200 MeV [21].

I thank my colleagues who have kindly permitted me to quote their work.

References

1. P.E. Hodgson, The Optical Model of Elastic Scattering, Clarendon Press, Oxford, 1963; Nuclear Reactions and Nuclear Structure, Oxford University Press, 1971; The Nucleon Optical Potential, World Scientific, 1995.
2. T. Tamura, *Rev. Mod. Phys.* **37**, 241, 1965.
   B. Buck, *Phys. Rev.* **130**, 712, 1963.
   B. Buck, A.P. Stamp and P.E. Hodgson, *Phil. Mag.* **8**, 1805, 1963.
3. P.E. Hodgson, *Contemporary Phys.* **31**, 295, 1990.
4. C. Mahaux, H. Ngô and G.R. Satchler, *Nucl. Phys.* **A449**, 354, 1986.
5. J.P. Delaroche and W. Tornow, *Phys. Lett.* **B203**, 4, 1988.
6. R.W. Finlay, J.R.M. Annand, J.R. Petler and F.S. Dietrich, *Phys. Lett.* **B155**, 313, 1985.
7. Su Zong Di and P.E. Hodgson, *J. Phys.* **G14**, 1485, 1988.

8. C.M. Perey and F.G. Perey, *Atom. Data and Nucl. Data Tables* **13**, 293, 1974; **17**, 1, 1976.

9. P.G. Young, Proceedings of the Specialists' Meeting on the Use of the Optical Model for the Calculation of Neutron Cross-Sections below 20 MeV. OECD, Paris, 127, 1986.

10. F.G. Perey, *Phys. Rev.* **131**, 745, 1963.

G.W. Greenlees, G.J. Pyle and Y.C. Tang, *Phys. Rev.* **171**, 1115, 1968.

11. C.M. Perey and F.G. Perey, *Phys. Rev.* **132**, 755, 1963.

12. P.E. Hodgson and E. Sartori, NEA Data Bank Report–198–U.INDC (NEA), 5, 1985.

13. A. Auce, R.F. Carlson, A.J. Cox, A. Ingemarsson, R. Johansson, P.U. Renberg, U. Sundberg, G. Tibell and R. Zorro, *Phys. Rev.* **C50**, 871, 1994.

14. T. Yamaya. Private communication, 1996.

15. M. Masaki and Y. Aoki, Annual Report of Tandem Accelerator Centre, University of Tsukuba, 20, 1992.

16. K. Amos. Private communication, 1996.

17. P. Demetriou, A. Marcinkowski and P.E. Hodgson, *Nucl. Phys.* **A596**, 67, 1996.

18. R. Bonetti, M.B. Chadwick, B.V. Carlson, M.S. Hussein and P.E. Hodgson, *Phys. Rep.* **202**, 171, 1991.

19. R. Bonetti, A.J. Koning, J.M. Akkermans and P.E. Hodgson, *Phys. Rep.* **247**, 1, 1994.

20. E. Gadioli and P.E. Hodgson, *Pre-Equilibrium Nuclear Reactions*, 1992.

21. H.B. Olaniyi, P. Demetriou and P.E. Hodgson, *J. Phys.* **G21**, 361, 1995.

P. Demetriou and P.E. Hodgson, *J. Phys.* **G22**, 1811, 1996.

A.A. Cowley, G.J. Arendse, J.W. Koen, W.A. Richter, J.A. Stander, G.F. Steyn, P. Demetriou, P.E. Hodgson and Y. Watanabe, *Phys. Rev.* **C54**, 778, 1996.

A.A. Cowley, G.J. Arendse, G.F. Steyn, J.A. Stander, W.A. Richter, S.S. Dimitrova, P. Demetriou and P.E. Hodgson. In preparation.