THEORETICAL MODELLING OF FUEL ASSEMBLY VIBRATIONS FOR VVER-TYPE REACTORS

V. Kinelev, S. Perov, V. Sulimov
Bauman Moscow State Technical University
(Russia)

Abstract

The present paper reviews several aspects of the problem of vibrational spectra interpretations for the VVER-type fuel assembly under normal and abnormal conditions. Theoretical models and appropriate software have been developed for dynamic analysis and eigencharacteristics extraction. An approach has been proposed toward the creation of simplified finite-element models of the fuel assemblies, which is based on the exception of the fuel pins from the model by allocation of the guide tubes with surrounding bunches of the fuel pins into substructures. Some results of eigencharacteristics computing and dynamic analysis are presented for the VVER-1000 fuel assembly. It is shown that a spectrum of eigenfrequencies depends on the clamping state of the guide tubes and the fuel pins in the spacing grids. The influence of reactor liquid filling on eigenfrequencies is evaluated analytically. The application of design sensitivity analysis and optimisation techniques to finite-element models tuning and anomaly detection is discussed.

Introduction

Noise diagnostics systems that use in-core instrumentation and ex-core neutron detectors are treated as components of monitoring and diagnostics systems of a reactor core. This raises the dual problems of noise signals interpretations and anomaly diagnosis. For PWR most of the resonant peaks in noise spectra are associated with mechanical vibrations both in the core and the whole together with a reactor core barrel and its individual parts – fuel assemblies and control elements. It should be pointed out that these peaks are most informative for diagnostics problems. For some years, quite a few studies have dealt with experimental and theoretical investigations in the field of noise analysis application for NPP structures monitoring. Some aspects of the modelling of core barrel vibrations for PWR have been presented in [1-3]. However, these studies pay less attention to fuel assemblies and control element vibrations. As a rule the models of the vibrations being used were rather simple [4,5].

Various applications of numerical methods for an identification of vibrational spectra of VVER-type fuel assemblies under both regular and abnormal conditions are considered in this work. The main task of this research is to define the influence of structural details and field conditions of fuel assemblies on their vibrational characteristics. This paper is a further development of approaches represented in [6].

The finite-element (FE) method is applied for modelling of vibrational behaviour of the fuel assemblies as one of the most universal methods of continual problems discretisation. Its important advantages are an invariance with respect to geometrical and mechanical features of modelling structures, relative simplicity of boundary conditions formulation and good computing characteristics of solving equations matrices. All this ensures the high efficiency of computer calculations and a simpler manipulation of structural parameters.

Anomalies such as mechanical damages of any elements, wear of supporting cups, degradation of a spring block and others can change the vibrational characteristics of fuel assemblies. Interactions between guide tubes, fuel pins and spacing grids can influence these characteristics as well. Such interactions are generally rather weak; under normal conditions fuel assemblies and guide tubes are practically disjointed in a dynamical sense with fuel pins and from each other. However, clamping in spacing grids can take place due to deformations of fuel assemblies, which can be caused by excessive axial force as a result of a discrepancy in assembly size, improper installation and/or defects of the spring block. Therefore, the computing of a strained state of the fuel assembly under axial forces is one more problem related to modelling. It should be noted that this problem presents independent interest, since the deformation of the fuel assembly can result in jamming of control elements inside guide tubes. Thus the mathematical models are needed to adequately describe all of the mentioned peculiarities.

The FE analysis includes static analysis of the strained state, calculations of eigencharacteristics and dynamical computations of forced vibrations parameters including action of axial forces. The applied problem-oriented software is available on PCs and workstations.

Finite-element models

The VVER-type fuel assembly represents a rather complex structure that consists of 19 guide tubes, 312 fuel pins, tail and head with spring block. Moreover the guide tubes and the fuel pins are connected to each other by means of 15 spacing grids. With such specifications, direct discretisation is unreasonable because models get too bulky and expensive for diagnostic purposes. Additionally, direct modelling of such regular structures yields many repeated and uninformative eigenfrequencies. These variables cause problems for computing, as well as for interpretation of results.

Therefore, it is necessary to develop approaches toward the creation of simplified FE models of fuel assemblies. The load-bearing structures of the fuel assemblies consist of the guide tubes, which are only connected to the tail and head. The fuel pins are not joined to the head and do not resist an axial load. The main problem in the model design is equivalent representation of interactions between the spacing grids and the guide tubes and fuel pins. One of the structural features of the VVER-type fuel assemblies is the existence of very small clearances in the spacing grids where fuel pins are practically disconnected from the load-bearing structure. Such a case is possible only under ideal conditions of fuel assemblies. In the event of misalignment or expanding of the fuel pins or deformations of the load-bearing structure, it is possible to clamp the fuel pins in the spacing grids and, hence, connect them to the load-bearing structure. This circumstance does not allow the exclusion of the fuel pins from theoretical models.

After structural analysis of the fuel assemblies followed the design of the base models which included only the load-bearing structures (guide tubes, spacing grids, tails and heads with spring blocks). This model takes into account the influence of the fuel pins on vibrational characteristics of assemblies by means of reducing stiffness and inertial characteristics of the guide tubes to be defined on the basis of substructure modelling consisting of one guide tube and the surrounding bunch of fuel pins. From these regular configurations can be extracted substructures consisting of one guide tube and six fuel pins for the central area of the fuel assembly and one guide tube and 22.5 fuel pins for the outlying area.

According to the FE model, the substructure of the guide tube and the bunch of fuel pins is comprised of 202 one-dimensional straight-line elements and 96 two-node spring elements. The base FE model of the fuel assembly consists of 619 one-dimensional straight-line elements and 305 two-nodes spring elements and has about 4000 degrees of freedom. One-dimensional straight-line elements simulate guide tubes, fuel pins and spacing grids. Two-node spring elements model elastic ties in spacing grids. As a result of their small size the head and tail of the fuel assembly are represented in the FE model by rigid bars. Two one-node spring elements on the tail and the head define boundary conditions.

In all of the above models it is assumed that the co-ordinates of one-dimensional elements nodes coincide with those of the end points of the modelling rods. Mechanical characteristics of an element are supposed to be defined if the type of material is specified. Since one-dimensional elements are tubular (having circular cross sections), their cross sectional outside diameters and wall thickness have to be identified. In any case, it is necessary to specify a length for each straight-lined element, a curvature radius of an axial line and an appropriate central angle for each curved element. The number of degrees of freedom in each node is equal to six. The stiffness matrix and the consistent mass matrix of a one-dimensional element possess a 12×12 dimension, but the stiffness matrix of a pointed spring element has a 6×6 dimension.

Finite-element analysis

Equations of movement of the elastic beam system modelling which result from the use of the FE method have the form:

$$M\ddot{u}+C\dot{u}+Ku=F, \tag{1}$$

where *M, C, K* are the reduced global mass, damping and stiffness matrices of FE model respectively; $\ddot{u},\dot{u},u$ are the vectors of accelerations, velocities and displacements of FE model nodes; *F* is the reduced vectors of external forces.

The response of the system on static load is defined from Eq. (1) after the elimination of terms including inertial and damping forces:

$$Ku = F \tag{2}$$

Eq. (1) represents a system of *n* differential equations of second order. Considering that, when the FE method discretisation is used, the degree of matrices in Eq. (1) is rather large, the routine procedures used for solving differential equations are ineffective. For this reason Newmark's β-method is applied for the integration of equation system (1).

The task of eigencharacteristics calculations is formulated in terms of a generalised algebraic eigenproblem:

$$K\phi - \xi M\phi = 0 \tag{3}$$

where (ξ,ϕ) is an eigenpair. An eigenvector is normalised by condition $\phi^T M\phi = 1$

The discretised eigenproblem (1) is solved by use of the subspace iteration method. The initial subspace is constructed on the basis of numeric characteristics investigations of the main diagonal elements of the matrices *K,M.* This method is designed to calculate the limited number of lower eigenpairs, including ones with algebraically multiple eigenvalues.

A special program has been developed for solving some of the diagnostics problems. The program is comprised of a set of routines intended for input data preparations, FE analysis, constraint definitions, sensitivity analysis and optimisation techniques, monitoring of iterations and convergence conditions, and presentation of results in graphical form. The program integrates procedures written in the FORTRAN and C++ languages and is oriented to platforms PC and workstations. The program is capable of solving the problems through the following means: eigenvalue extraction, static and dynamic analysis of structures under thermal and force load, design sensitivity analysis and FE models tuning.

Eigencharacteristics extraction

Here, we discuss some results of the eigencharacteristics computations for the fuel assemblies of VVER-1000 NPP. At the fist stage the substructure, including one guide tube and the six surrounding bunch of fuel pins, was investigated. It is assumed that the guide tube is fixed at the tail and the head and the fuel pins are fixed in the tail and are connected to the guide tube by the spacing grids. The fuel pins represent a circular shell filled with nuclear fuel made in the shape of tablets. Therefore, it is assumed that stiffness characteristics are due only to the pins' shell but, when inertial characteristics are defined, it is also necessary to take into account the mass of the fuel tablets.

A number of computations have been carried out with various stiffness characteristics of spring element simulating elastic ties in the spacing grids. Consider some limited cases. Obviously, that there's no point in computations without ties in spacing grids, because in such cases the vibrations of the fuel pins and the guide tube are completely independent.

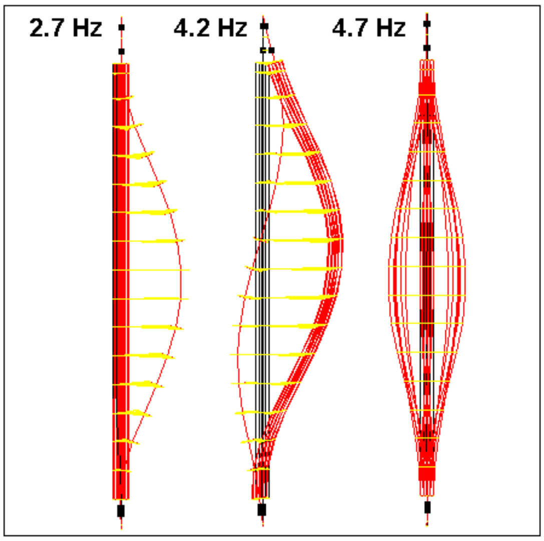
In the first case it is assumed that all fuel pins are fixed in the spacing grids in lateral directions (X, Y) only and free in longitudinal direction (Z). In this case the eigenfrequencies of the three lowest modes are equal to 1.5, 4.2 and 8.4 Hz. For comparison, the eigenfrequencies of the three lowest modes of the single guide tube without fuel pins are equal to 4.7, 12.9 and 25.3 Hz. It is possible to calculate the reducing stiffness and inertial characteristics of an equivalent beam. Considering that the shapes of the guide tube and fuel pin modes are similar, the reducing characteristics of the equivalent beam are close to total ones for every element.

In the second case it is supposed that all fuel pins are fixed in each of three directions (X, Y, Z). The eigenfrequencies of the three lowest modes are equal to 2.3, 6.1 and 11.2 Hz. In this situation it is also possible to calculate the reducing characteristics of the equivalent beam. Moreover the inertial characteristics must correspond to the total ones of the substructure and the stiffness characteristics have to be adjusted such that the eigenfrequencies of the equivalent beam and the substructure are in agreement.

The two instances discussed above describe limiting cases. Such procedures have also been performed for some intermediate cases, and it has been found necessary to adjust the characteristics of equivalent beams. After this initial examination, FE analysis of the fuel assembly as a whole is carried out. The stiffness and inertial characteristics of the guide tubes in FE model are taken to equal ones of considered substructures in dependence of clamping state in the spacing grids. Because the spacing grids are clamped only on the central guide tube, it is necessary to once more consider the various clamping states in the outlying area. In addition, the different boundary conditions can be realised during operation cycles. In this paper the results are presented for a normal situation only; that is, when boundary conditions on both ends of the fuel assembly are close to free supports.

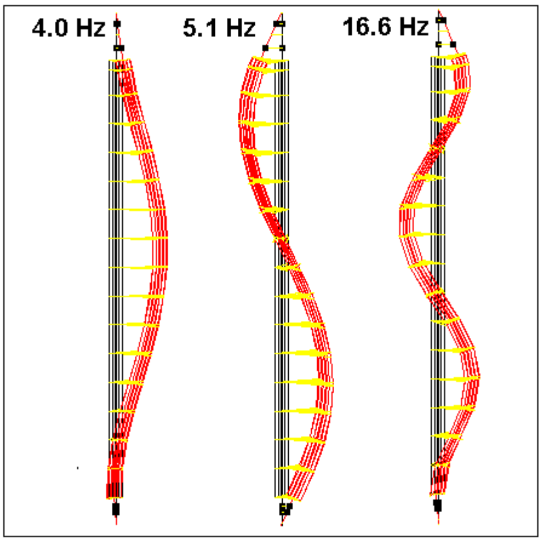
In the first case, an ideal condition of the fuel assembly is considered in which there are clearances in the spacing grids or interactions are rather weak. The eigenfrequencies of the three lowest modes in such case are equal to 2.7, 4.2 and 4.7 Hz. The corresponding shapes of the modes are shown in Figure 1. The first mode corresponds to vibrations of the central guide tube together with spacing grids as the fist shape. The second mode is associated with joint movements of the central tube as second shape and other tubes as the first shape. A number of next modes with the same eigenfrequencies are characterised by movements of the guide tubes in the outlying area according to the first shape.

Figure 1. Shapes of modes (case 1)



In the second case it is supposed that all guide tubes are fixed to the spacing grids only in a lateral direction and the fuel pins' interaction with the spacing grids is very weak. The eigenfrequencies of the three lowest modes are equal to 4.0, 5.1 and 16.6 Hz. The corresponding mode shapes are shown in Figure 2. In this case the structure of the fuel assembly is coupled, which can be observed from the mode shapes.

Figure 2. Shapes of modes (case 2)



In the third case it is assumed that all guide tubes and all fuel pins are fixed in the spacing grids in lateral directions only, and in forth case, that all guide tubes and fuel pins are fixed in all directions. The corresponding eigenfrequencies are equal to 1.0, 1.7, 5.0 Hz and 2.0, 5.1 and 8.3 Hz respectively. The mode shapes for these cases are close to those displayed in Figure 2.

Thus, the clamping of guide tubes and fuel pins in spacing grids have a large influence on the eigenfrequencies. One of the resulting problems is to define the dependence of eigencharacteristics spectra on the real clamping state in the spacing grids. Moreover, fuel assemblies with different clamping states in the spacing grids can simultaneously exist in the core. This leads to the appearance of a wide band of eigenfrequencies peaks (1.0 - 4.7 Hz) according to the first shape in spectra measuring by ex-core neutron detectors or ex-core vibrational sensors. Such results are often observed in experiments.

When this model is applied toward the diagnostics of fuel assembly conditions, aside from the cases mentioned above it is possible to vary parameters such as the stiffness characteristics of supports and elements, whose changing can result from failures in the head or the tail, in a support cup or with guide tube failure.

The mathematical model to be considered does not take into account the influence of reactor liquid filling on fuel assembly dynamics. It is known that liquid filling leads to the formation of an associated mass of liquid and, hence, to decrease in the eigenfrequencies. Corresponding effects on eigenfrequencies can be analytically evaluated by use of Stokes' formula for calculation of an associated mass of liquid when an endless cylinder oscillates in bounded cavity:

$$m_1 = \pi \rho r^2 \frac{R^2 + r^2}{R^2 - r^2} \tag{4}$$

where m_1 is associated mass of liquid, ρ is liquid density, r is radius of cylinder, and R is radius of cavity.

It is possible to use more exact half-empirical dependencies that take into account mutual locations of cylinders. These evaluations give a maximal value of associated mass $m_1 / m = 0.25$, where m is mass of cylinder. This leads to a maximal decrease of the eigenfrequencies by 12%. Such a difference is within the limits of accuracy of the model if an allowance is made for unreliable initial data. A small influence of reactor liquid filling on the eigenfrequencies is confirmed by experiments. As one of problems is tuning of the FE model by using experimental data, it is not necessary to complicate the theoretical model by including the liquid filling variable.

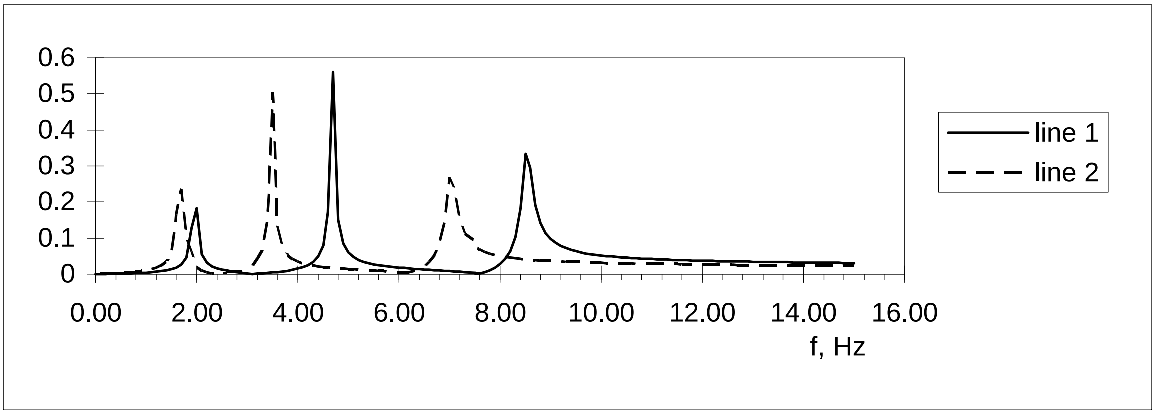
Dynamical analysis

The computation of forced vibrations allows the most complete interpretation of experimental vibrational spectra. The adequacy of computing results and experimental data depends on the following factors. It is necessary to develop the models of exited forces. The basis for such models can be experimental data on levels and spectra of pressure oscillations and mechanical vibrations in a reactor vessel. In addition the damping coefficients must be known. Unfortunately, it is impossible to obtain such data through theoretical methods, and consequently reliable experimental data is needed.

For testing methods and software the computations have been performed for pseudo random excitations in a specified frequency band. In such cases the peaks in vibrational spectra correspond to eigenfrequencies. Therefore this method can be applied to computations of eigenfrequencies when external static forces affect the fuel assembly (including axial forces).

As dynamical analysis requires a large amount of computing time, the reduced FE models are used. It is supposed that the axial forces deform the fuel assembly and therefore all guide tubes and fuel pins are fixed in the spacing grids. The computing spectra for this case are shown in Figure 3. Line 1 corresponds to the vibration of the fuel assembly without axial forces. The peaks in the spectrum correspond to eigenfrequencies 2.0, 4.6 and 8.3 Hz. Line 2 is obtained when axial compressing force equal 25 kN affect at the fuel assembly head. This figure indicates that the eigenfrequencies decrease under axial compressing force when this force is sufficiently large and the fuel assemblies are fixed in the spacing grid.

Figure 3. Vibrational spectra without (1) and with (2) axial force



Under actual conditions, if axial force is not sufficient to cause large deformations in the fuel assembly and full clamping in the spacing grids, its influence can be exhibited through altering the clamping state in the spacing grids.

Identification of anomalies

An application-oriented approach to the problem of identification of structural anomalies is proposed in this section. The main dynamical characteristics of the structure are considered as continuous functions of the bounded set of design variables. The occurrence of anomalies such as mechanical damage to the structure can be detected as a result of changes in structural dynamical characteristics (e.g. natural frequencies or eigenvalues) caused by changes in design variables. It is clear that nominal (without anomalies) and anomalous structures are characterised by different spectra. Thus the objective is to simultaneously minimise individual differences between corresponding eigenvalues of the nominal (or initial) and the anomalous structures under consideration. These differences can be characterised by the following set of error criteria:

$$f_i(x) = \left|\zeta_i(x) - \zeta_i^a(x)\right|, \quad x \in \mathrm{X} \subset R^n, \quad i = \overline{1,N}$$

where *x*, *X* is the vector of design variables and its feasible domain; R^n is n-dimensional Euclidean space; $\zeta_i(x) - \zeta_i^a(x)$ are eigenvalues that belong to nominal and anomalous spectra correspondingly; *N* is the number of eigenvalues to be considered.

Further it is suggested that the essential data both on nominal and anomalous structural spectra are sufficiently complete and that the FE model of the structure is able to simulate the anomalies adequately. The solution procedure involves a rational varying of design parameters with the aim of obtaining the desired spectral properties of the model. The problem of tuning the current spectrum using data on the anomalous model leads to an optimisation problem that can be formalised as follows:

$$\min_{x \in X \subset R^n} f(x),$$

where

$$f(x) = \left(f_1(x),\ldots,f_N(x)\right)^T$$

It should be noted that here the objective mapping is bounded from below by 0, i.e.:

$$0 \le \left|\zeta_i(x) - \zeta_i^a(x)\right| \quad \forall x \in X \subset R^n, \quad i = \overline{1,N}$$

It can be shown that following certain assumptions this problem is equivalent to the following vector optimisation problem:

$$min\ (\lambda_1,\ldots,\lambda_N)^T$$

subject to constraints

$$\begin{aligned} &(x,\lambda) \in X \times R^N; \\ &-\lambda_i \le \zeta_i(x) - \zeta_i^a(x) \le \lambda_i, \quad i = \overline{1,N} \end{aligned}$$

Numerical solutions of the vector optimisation problem can be obtained by use of such methods as scalarisation techniques and bicriterial algorithms.

One more general formulation of the problem under consideration can be written as follows:

$$\min_{x \in X \subset R^n} \max \left\{f_1(x),\ldots,f_N(x)\right\}$$

So the general optimisation problem is to find the vector of design parameters x = $(x_1,\ldots,x_n)^T$ which brings the minimum to the objective function $f(x) = \max\{f_1(x),\ldots,f_N(x)\}$ over the feasible design space $X \subset R^n$. This problem is solved using a recoursive quadratic programming technique with an active set strategy. In order to overcome inherent difficulties associated with non-differentiability of the min-max type problems, an effective smoothing-out procedure is introduced. The corresponding smoothing approximations preserve such essential properties of functions $f_i(x)$ as convexity and continuous differentiability. This approach provides a smoothing mechanism for multidimensional general non-linear functions in cases where locations of peculiarities are unknown. It should be noted that in general different sets of design parameters may produce the same combination of structural spectral properties. Consequently, it is important to determine a subset of design variables exerting the greatest influence on the structural properties to be corrected. It is also necessary to choose a sufficiently large number of eigenvalues to be analysed.

The approach proposed here is applied to a problem of tuning the spectrum of a simplified model of the fuel assembly. Only vibrations in the plane OYZ are considered. The lowest three eigenvalues that correspond to the natural frequencies (2.0, 4.6, and 8.3 Hz) of the nominal (without anomalies) structure are 151.630, 852.160, and 2748.11. Then the following anomalies are imposed to the structure: stiffness characteristics of the lower support and of the spring element are reduced up to 9.0% and 2.5% respectively. Now the lowest three eigenvalues that correspond to the natural frequencies (1.87, 4.20, and 7.811 Hz) of the structure with anomalies are 138.438, 695.418, and 2408.47. The problem is to tune the current spectrum of the structure using data on the anomalous spectrum by varying the design parameters x_1, x_2, x_3 representing stiffness characteristics of the lower support, the upper support and the spring correspondingly. The iteration history is shown in Figures 4 and 5. The initial value of the objective function is 339.63; after ten iterations this value is 0.273. It should be pointed out that the same result is achieved by varying of only two parameters x_1, x_3, whose resulting values after ten iterations are 10.5% and 2.5% respectively. Therefore the tuning process denotes the design variables with values rather close to those of the parameters describing the imposed anomalies.

Figure 4. Iterative refinement of design variables

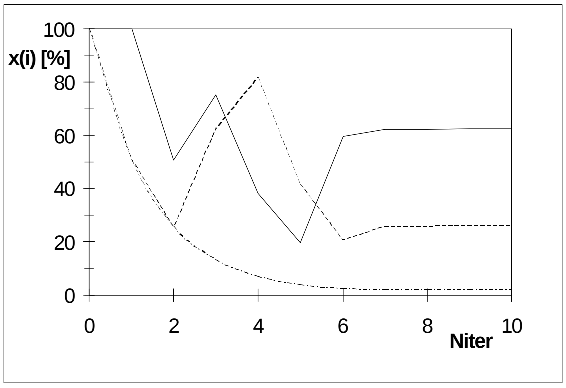
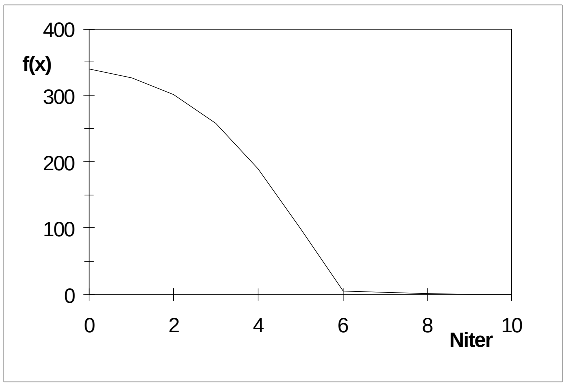


Figure 5. Convergence of the objective function



This numerical example is demonstrated to be a principal possibility toward the identification of anomalies through their formulation as optimal problems. The approach proposed in this paper is therefore a perspective for actual use in the diagnosis of technical anomalies in various structures.

Conclusions

Mathematical modelling methods have been applied to the problem of vibrational spectra identifications. The results can be used for planning experiments, interpreting experimental data and anomaly diagnosis.

The considered models need to be further adjusted by vibration experiments both at the original NPP and test benches.

The above models and software may be considered as essential components of the expert system being developed for the VVER-type NPP.

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